Flow helicity: could it be useful for fusion?

Y. Kosuga[1,2]

1. Research Institute for Applied Mechanics, Kyushu Univ.
2. Research Center for Plasma Turbulence, Kyushu Univ.

In collaboration with: M. Sasaki, Z.B. Guo

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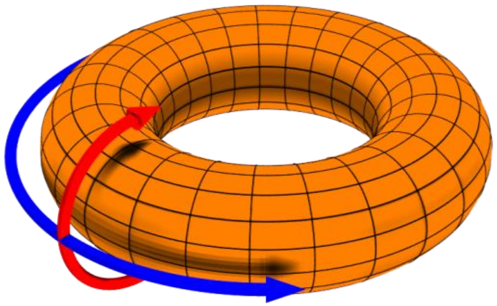
Outline

- Introduction and motivation
  - 3D flow pattern
  - Helicity as an important parameter
- Fluctuation helicity
  - How reflectional symmetry broken in magnetized plasmas
- Helicity balance in turbulent plasmas
  - Analysis on numerical data, NLD
- Application to large scale flows and fields
- Discussion and Conclusion

Introduction

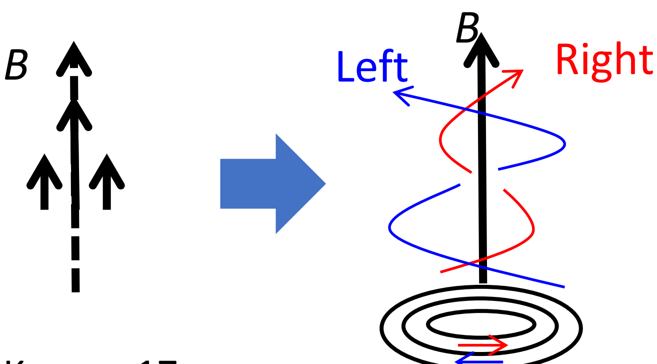
- Flows are an important element for fusion plasmas



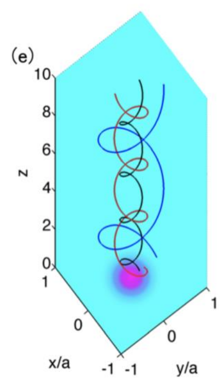
- ✓ ExB flows
- ✓ Toroidal flows

➢ Naturally leads to 3D flow patterns

- 3D flow patterns are generated by turbulence



Kosuga 17



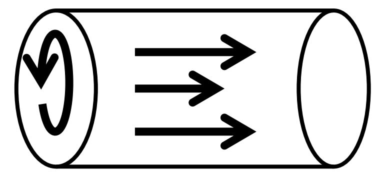
Sasaki 17

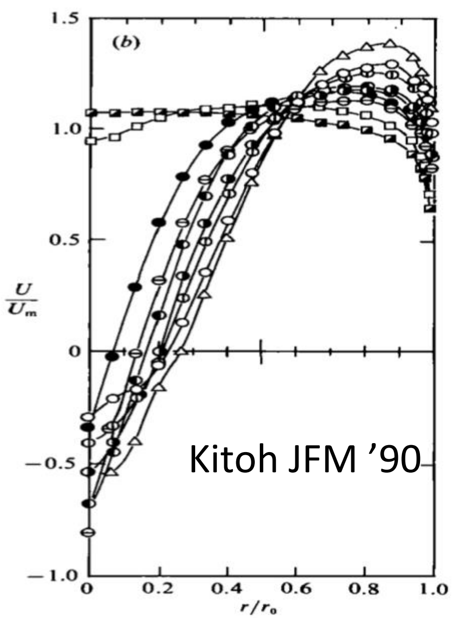
➢ How quantify the geometry/topology of 3D flows?

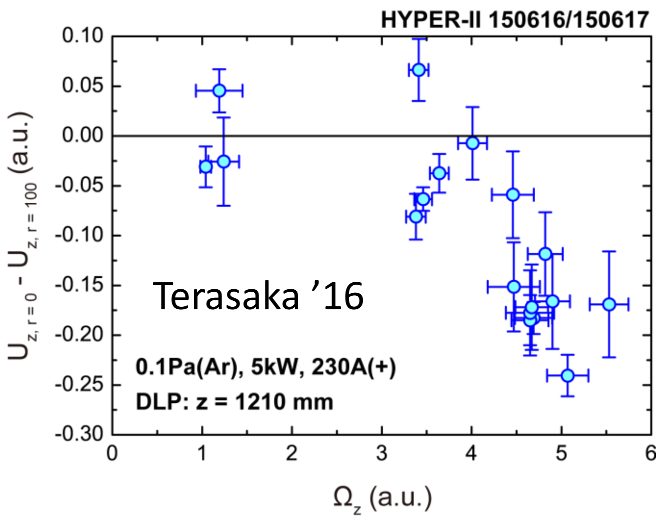
helicity $\mathbf{v} \cdot \omega$

Helicity can be important for plasmas

- 3D flows excited
- Impact on large scale flows/fields
  - ➢ Dynamo, B-field Moffatt '78
  - ➢ Momentum transport, flows in a pipe



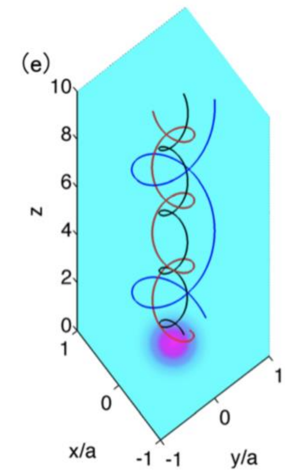


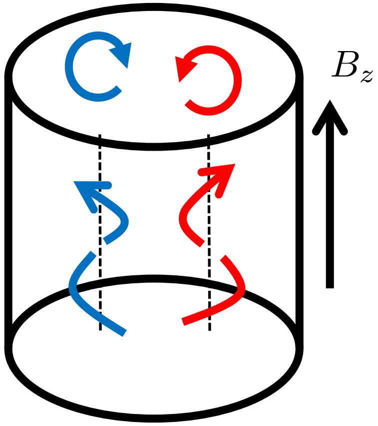


Helicity for fusion plasmas?

Helicity likely finite for turbulent plasmas

- Large scale flows -> maybe yes
  - Zonal flows, intrinsic parallel flow, PVG -> ZF, ...
- Fluctuation -> very likely finite
  - Plasma turbulence is 'wavy'
- Turbulent plasmas -> gas of helices
- May merit some analysis for firmer basis
  - Key mechanism? Relevant parameter?
  - Application?





3D models required for having helicity

$$\mathbf{v} = \frac{c}{B}\hat{z} \times \nabla\phi + v_z\hat{z} \qquad \omega = \nabla \times \mathbf{v}$$

- For mean field

$$\langle \mathbf{v} \rangle \cdot \langle \omega \rangle = \langle v_\theta \rangle' \langle v_z \rangle$$

Vorticity $\langle \omega_z \rangle = \langle \nabla_\perp^2 \phi \rangle$

Flow $\langle v_z \rangle$

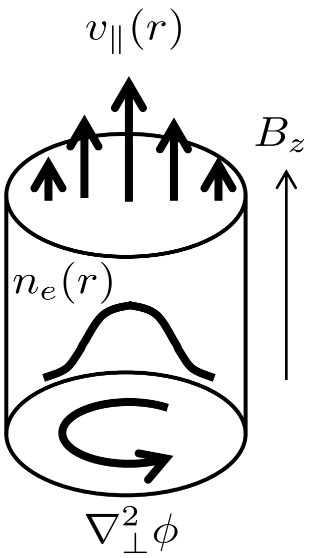
- For fluctuation

$$\langle \tilde{v} \cdot \tilde{\omega} \rangle = 2\frac{c}{B_z}\langle \tilde{v}_z \nabla_\perp^2 \tilde{\phi} \rangle + 2\frac{c^2}{B_z^2}\langle \partial_y \tilde{\phi} \partial_{zx} \tilde{\phi} \rangle - \partial_x \frac{c}{B_z}\langle \tilde{v}_z \partial_x \tilde{\phi} \rangle$$

|| coupling $\tilde{v}_z$ ∂_z

➢ 3D models required

Model



Reduced fluid model

$$(\partial_t + \mathbf{v}_{E\times B}\cdot\nabla)\rho_s^2\nabla_\perp^2\frac{e\phi}{T_e} = D_\parallel \nabla_z^2\left(\frac{\tilde{n}_e}{\langle n\rangle} - \frac{e\tilde{\phi}}{T_e}\right) - \nu_d\rho_s^2\nabla_\perp^2\frac{e\phi}{T_e} + \mu_\Omega\nabla_\perp^2\rho_s^2\nabla_\perp^2\frac{e\phi}{T_e}$$

$$(\partial_t + \mathbf{v}_{E\times B}\cdot\nabla)n_e + \langle n\rangle\nabla_z v_z = D_\parallel\nabla_z^2\left(\frac{\tilde{n}_e}{\langle n\rangle} - \frac{e\tilde{\phi}}{T_e}\right) + \mu_N\nabla_\perp^2 n_e$$

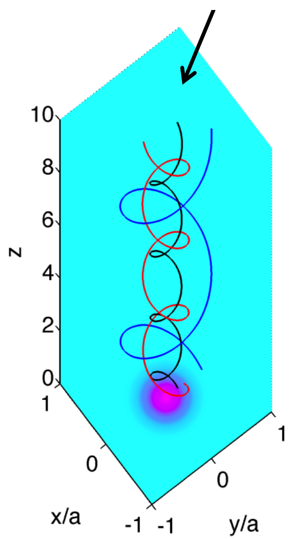
$$(\partial_t + \mathbf{v}_{E\times B}\cdot\nabla)v_z = \frac{e}{m_i}E_z - \nu_d v_z + \mu_v\nabla_\perp^2 v_z$$

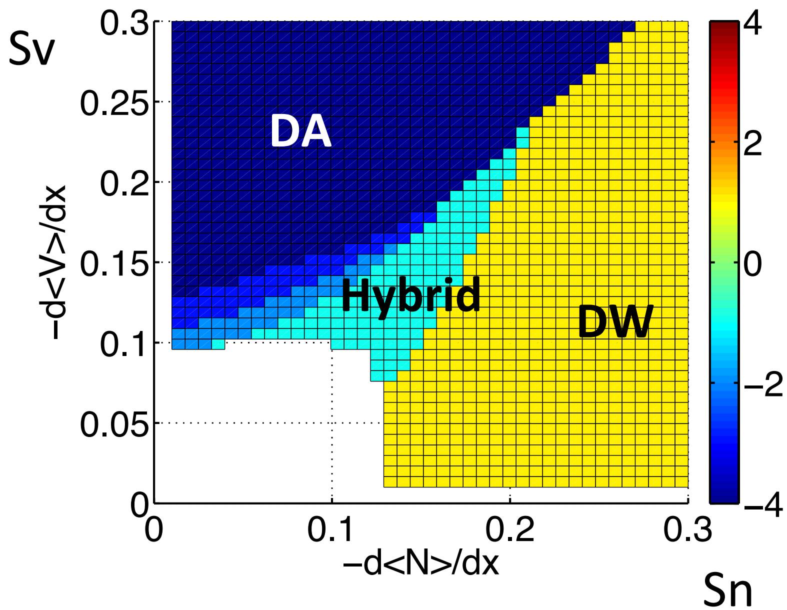
- Without flows -> drift waves (Hasegawa-Wakatani/Mima)
- With flows -> Drift waves + $\nabla v_\parallel$ driven mode (PVG turb., (D'Angelo))
- Linear dispersion: $\omega = \frac{\omega_{*e}}{1+\rho_s^2 k_\perp^2}\frac{1+\sqrt{D}}{2}$ $\quad D = 1-(1+k_\perp^2\rho_s^2)M^2$ $\quad M = \frac{\langle v_\parallel\rangle'}{c_s/L_n}$
- Better version is implemented in NLD code

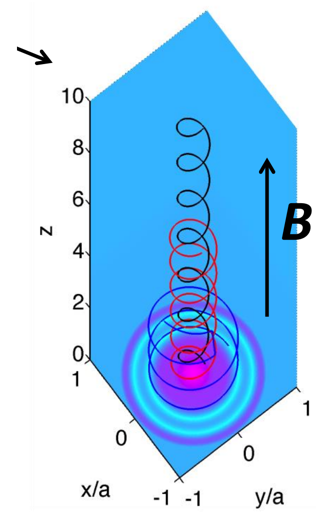
NLD code

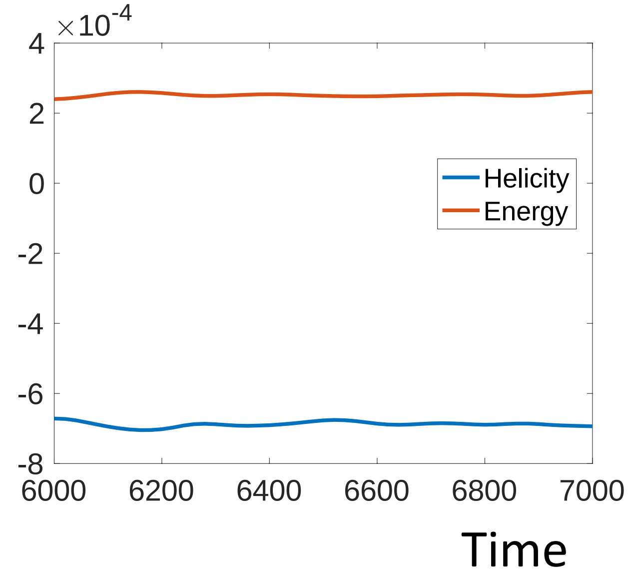
Sasaki, Kosuga, et al., Phys. Plasmas **24** 112103 (2017)

- Fluid code, density, vorticity, parallel flow
- Source driven 256 points in r -16<m<16 -16<n<16
- Various instabilities, flow patterns obtained









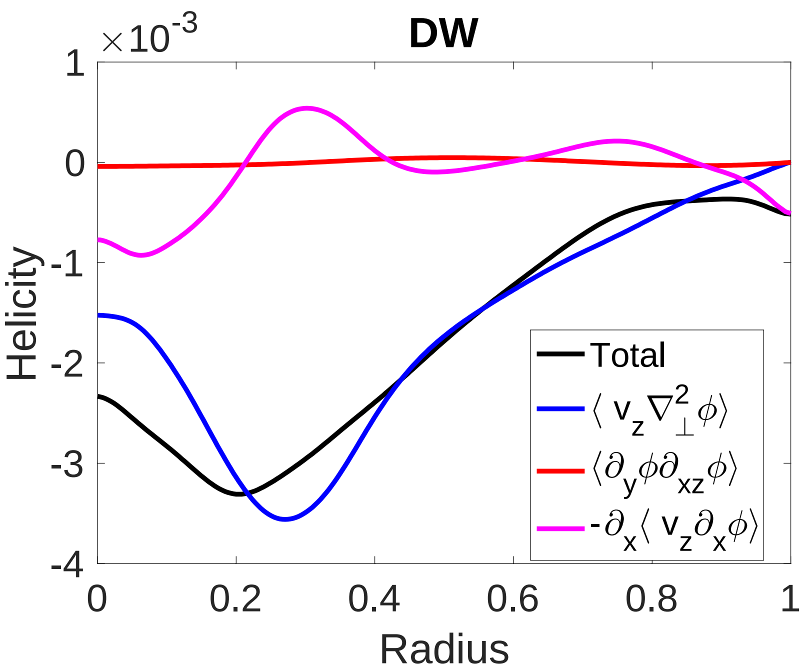
Fluctuation helicity

$$\langle \tilde{v} \cdot \tilde{\omega} \rangle = 2\frac{c}{B_z}\langle \tilde{v}_z \nabla_\perp^2 \tilde{\phi} \rangle + 2\frac{c^2}{B_z^2}\langle \partial_y \tilde{\phi} \partial_{zx} \tilde{\phi} \rangle - \partial_x \frac{c}{B_z}\langle \tilde{v}_z \partial_x \tilde{\phi} \rangle$$

Magnitude: $\frac{k_z c_s}{\omega}\rho_s^2 k_\perp^2 \left|\frac{e\phi}{T_e}\right|^2$ $\quad \frac{k_z c_s}{\omega_{ci}}\rho_s^2 k_x k_y \left|\frac{e\phi}{T_e}\right|^2$ $\quad \frac{k_z c_s}{\omega}\rho_s^2 k_x \frac{1}{L_I}\left|\frac{e\phi}{T_e}\right|^2$

Dominant

Numerically:



$$\langle \tilde{v} \cdot \tilde{\omega} \rangle \cong 2\frac{c}{B_z}\langle \tilde{v}_z \nabla_\perp^2 \tilde{\phi} \rangle$$

- ‘Equipartitioned’

$$\langle \tilde{v}_\perp \cdot \tilde{\omega}_\perp \rangle \cong \langle \tilde{v}_z \tilde{\omega}_z \rangle$$

- May simplify measurement

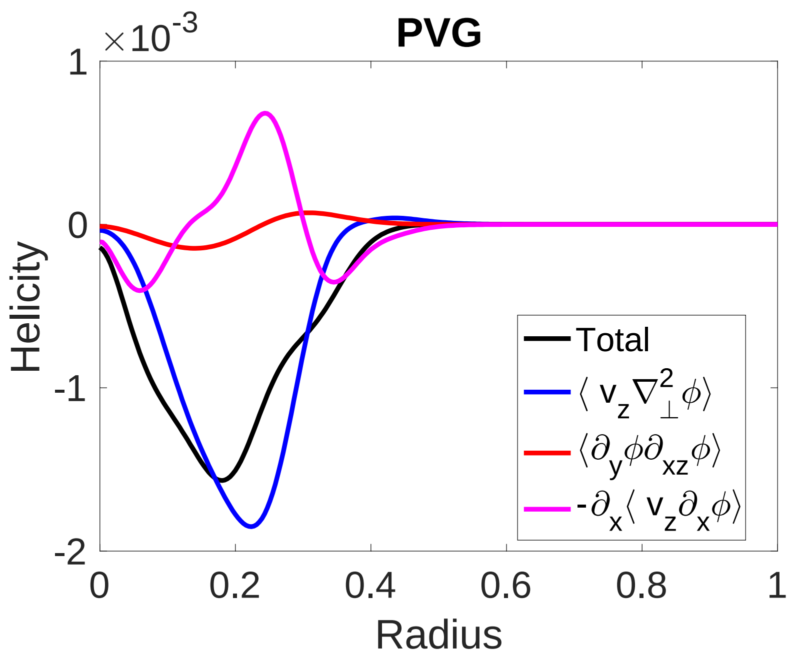
Fluctuation helicity

$$\langle \tilde{v} \cdot \tilde{\omega} \rangle = 2\frac{c}{B_z}\langle \tilde{v}_z \nabla_\perp^2 \tilde{\phi} \rangle + 2\frac{c^2}{B_z^2}\langle \partial_y \tilde{\phi} \partial_{zx} \tilde{\phi} \rangle - \partial_x \frac{c}{B_z}\langle \tilde{v}_z \partial_x \tilde{\phi} \rangle$$

Magnitude: $\frac{k_z c_s}{\omega}\rho_s^2 k_\perp^2 \left|\frac{e\phi}{T_e}\right|^2$ $\frac{k_z c_s}{\omega_{ci}}\rho_s^2 k_x k_y \left|\frac{e\phi}{T_e}\right|^2$ $\frac{k_z c_s}{\omega}\rho_s^2 k_x \frac{1}{L_I} \left|\frac{e\phi}{T_e}\right|^2$

Dominant

Numerically:



$$\langle \tilde{v} \cdot \tilde{\omega} \rangle \cong 2\frac{c}{B_z}\langle \tilde{v}_z \nabla_\perp^2 \tilde{\phi} \rangle$$

- ‘Equipartitioned’

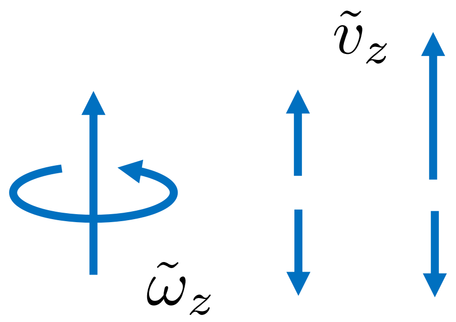
$$\langle \tilde{v}_\perp \cdot \tilde{\omega}_\perp \rangle \cong \langle \tilde{v}_z \tilde{\omega}_z \rangle$$

- May simplify measurement

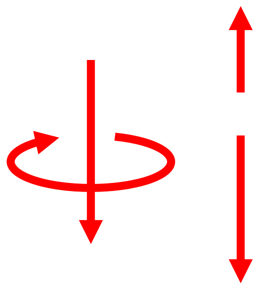
Fluctuation helicity

$$\langle \tilde{v} \cdot \tilde{\omega} \rangle \cong 2\frac{c}{B_z}\langle \tilde{v}_z \nabla_\perp^2 \tilde{\phi} \rangle = 2\langle \tilde{v}_z \tilde{\omega}_z \rangle$$

- // symmetry breaking is a key for finite value

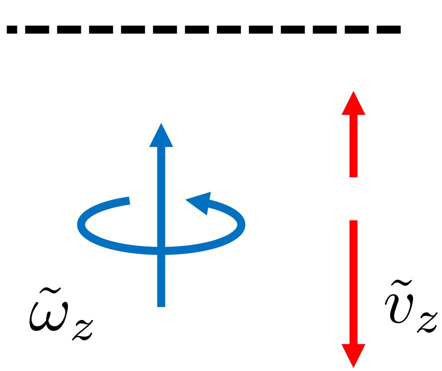


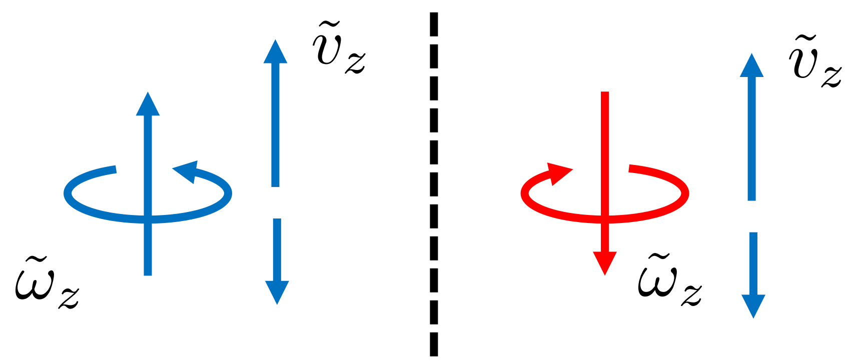
c.f.



$$\phi \to -\phi$$
$$\tilde{\omega}_z \to -\tilde{\omega}_z$$
$$\tilde{v}_z \to -\tilde{v}_z$$

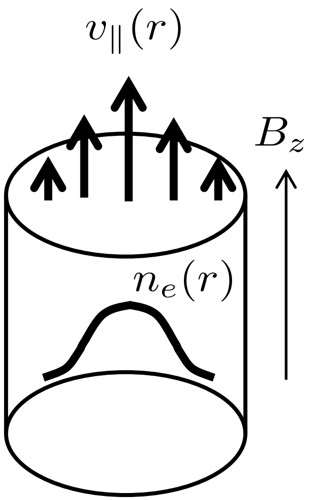
- Reflectional symmetry likely broken





An example: PVG mode

$$\langle \tilde{v} \cdot \tilde{\omega} \rangle \cong 2\frac{c}{B_z}\langle \tilde{v}_z \nabla_\perp^2 \tilde{\phi} \rangle \propto k_z(-k_\perp^2)$$



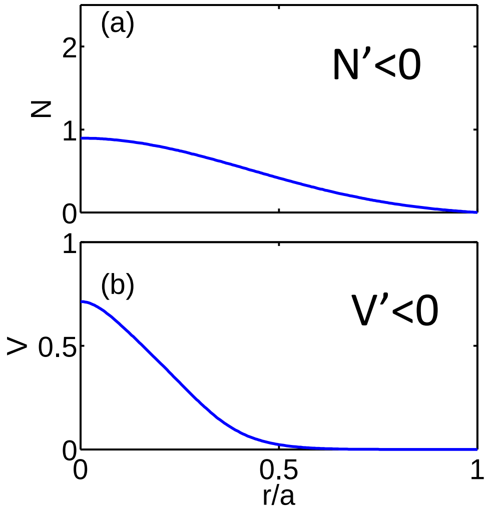
- Several candidates
  - DW + // symmetry breaking
  - PVG mode
- PVG (parallel velocity gradient) mode
  - NBI plasmas, SOL, ITB, or space plasmas
  - Dispersion relation: $(1+\rho_s^2 k_\perp^2)\omega^2 - \omega_{*e}\omega - k_z^2 c_s^2 \left(1 - \frac{k_y \langle v_z \rangle'}{k_z \omega_{ci}}\right) = 0$
- Preferred wave number
  - ✓ Necessary condition: $k_z k_y \langle v_z \rangle' > 0$
  - ✓ Most unstable wave number $k_z = k_y \frac{\langle v_z \rangle'}{2\omega_{ci}}$

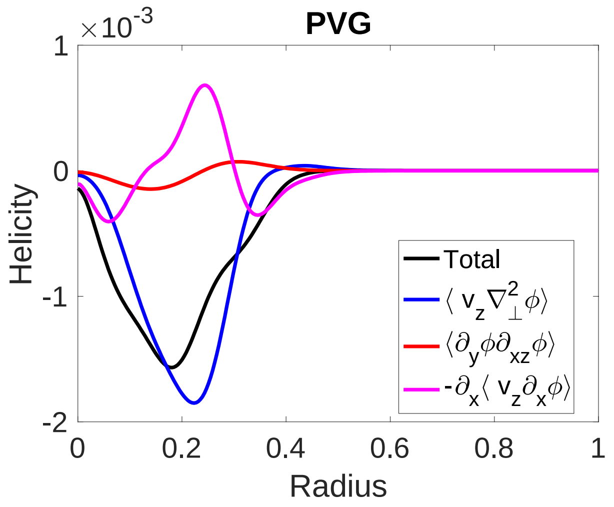
Helicity for PVG mode

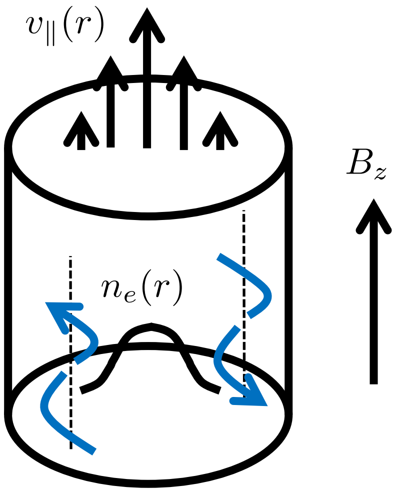
$$\langle \tilde{\mathbf{v}} \cdot \tilde{\omega} \rangle = 4 c_s \omega_{ci} \frac{c_s / L_n}{\langle v_z \rangle'} \sum_{\mathbf{k}} k_\perp^2 \rho_s^2 \left| \frac{e \tilde{\phi}_{\mathbf{k}}}{T_e} \right|^2 \propto -\langle n \rangle' \langle v_z \rangle' B_z$$

- In the simulation:

What we have:







vω<0

Can flip helicity?

Helicity balance

- Helicity balance

$$\partial_t \left[\langle \tilde{\mathbf{v}} \cdot \tilde{\omega} \rangle - 2 \frac{eB_z}{m_i c} \left\langle \tilde{v}_z \frac{e\tilde{\phi}}{T_e} \right\rangle - 2 \frac{c^2}{B_z^2} \left\langle \tilde{E}_y \partial_z \tilde{E}_x \right\rangle + \partial_x \langle \tilde{v}_y \tilde{v}_z \rangle \right]$$

$$+ \partial_x \left[-2 \frac{eB_z}{m_i c} \langle \tilde{v}_x \tilde{q} \tilde{v}_z \rangle + 2 \frac{eB_z}{m_i c} \frac{c^2}{B_z^2} \left\langle \tilde{E}_x \tilde{E}_z \right\rangle + 2 \langle v_y \rangle' \langle \tilde{v}_x \tilde{v}_z \rangle \right]$$

Spatial flux

$$+ 2 \langle v_z \rangle' \partial_x \langle \tilde{v}_x \tilde{v}_y \rangle - 2 \langle v_y \rangle' \partial_x \langle \tilde{v}_x \tilde{v}_z \rangle - 2 \frac{eB_z}{m_i c} \frac{\langle n \rangle'}{\langle n \rangle} \langle \tilde{v}_x \tilde{v}_z \rangle - 2 \frac{eB_z}{m_i c} \frac{\Gamma_n}{\langle n \rangle} \langle v_z \rangle'$$

$$= -2 \frac{eB_z}{m_i c} \left(\langle \tilde{v}_z \mu_n \nabla_\perp^2 \frac{e\tilde{\phi}}{T_e} \rangle - \nu \langle \tilde{v}_z \frac{e\tilde{\phi}}{T_e} \rangle + 2\nu \langle \tilde{v}_z \rho_s^2 \nabla_\perp^2 \frac{e\tilde{\phi}}{T_e} \rangle \right)$$

Relaxation, Transport

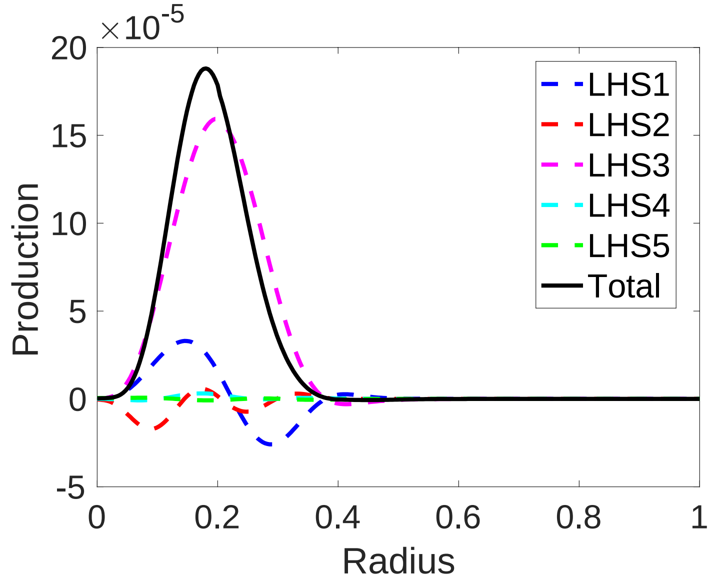
Dissipation

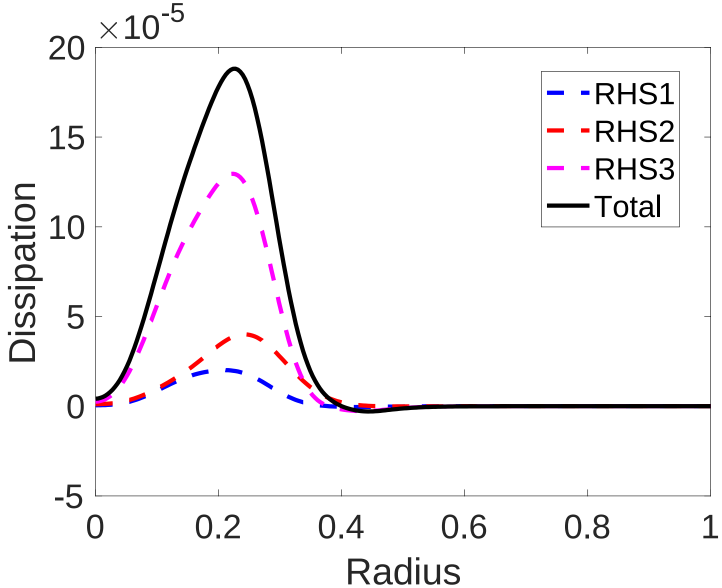
- Steady state confirmed both for energy and helicity
- How well source/sink balance satisfied?

Helicity balance for PVG

$$\int dx \left[\langle v_z \rangle' \partial_x \langle \tilde{v}_x \tilde{v}_y \rangle - \langle v_y \rangle' \partial_x \langle \tilde{v}_x \tilde{v}_z \rangle - \frac{eB_z}{m_i c} \frac{\langle n \rangle'}{\langle n \rangle} \langle \tilde{v}_x \tilde{v}_z \rangle - \frac{eB_z}{m_i c} \frac{\Gamma_n}{\langle n \rangle} \langle v_z \rangle' + \frac{eB_z}{m_i c} \frac{e}{m_i} \frac{\langle \tilde{E}_z \tilde{n} \rangle}{\langle n \rangle} \right]$$

$$= \int dx \left[-\frac{eB_z}{m_i c} \left(\langle \tilde{v}_z \mu_n \nabla_\perp^2 \frac{\tilde{n}}{\langle n \rangle} \rangle - \nu \langle \tilde{v}_z \frac{\tilde{n}}{\langle n \rangle} \rangle + 2\nu \langle \tilde{v}_z \rho_s^2 \nabla_\perp^2 \frac{e\tilde{\phi}}{T_e} \rangle \right) \right]$$





- Global balance: LHS = 6.3505x10^{-4} RHS = 8.4994x10^{-4}

- Relevant terms

$$\int dx \nu \langle \tilde{\mathbf{v}} \cdot \tilde{\omega} \rangle \sim \int dx \frac{eB_z}{m_i c} \frac{\langle n \rangle'}{\langle n \rangle} \langle \tilde{v}_x \tilde{v}_z \rangle$$

Helicity balance for PVG

- Dominant balance

$$\int dx \nu \langle \tilde{\mathbf{v}} \cdot \tilde{\omega} \rangle \sim \int dx \frac{eB_z}{m_i c} \frac{\langle n \rangle'}{\langle n \rangle} \langle \tilde{v}_x \tilde{v}_z \rangle$$

  - Non-perturbative relation

- Helicity <-> driving flux (momentum flux)

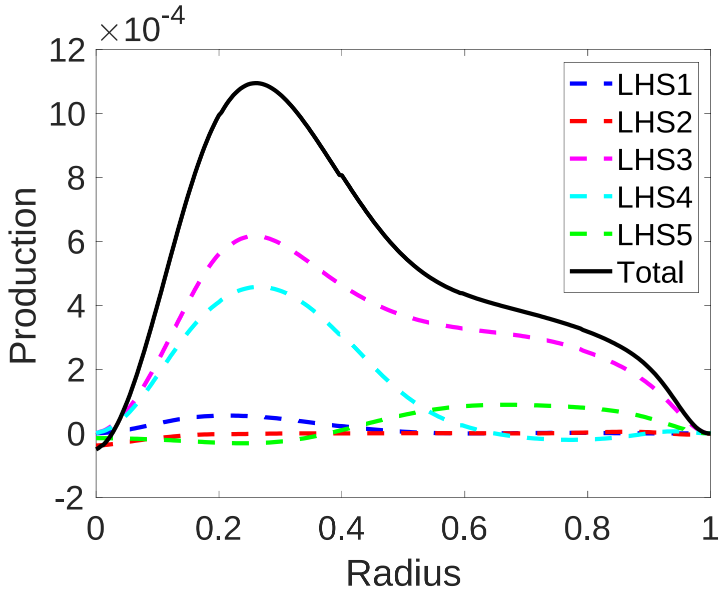
  - Finite flux <-> // symmetry breaking necessary $\propto k_z k_y$
  - Production $-\langle \tilde{v}_x \tilde{v}_z \rangle \langle v_z \rangle' > 0$

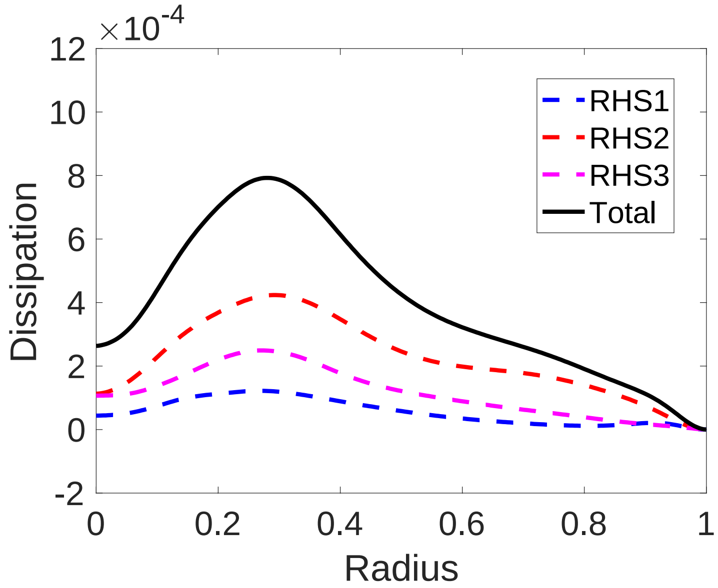
- Sign: $\langle \tilde{v} \cdot \tilde{\omega} \rangle \propto -B_z \langle n \rangle' \langle v_z' \rangle$

  - Consistent with mode analysis
  - Hold both for linear/nonlinear regime, a robust feature

Helicity balance for DW

$$\int dx\left[\langle v_z\rangle'\partial_x\langle\tilde v_x\tilde v_y\rangle-\langle v_y\rangle'\partial_x\langle\tilde v_x\tilde v_z\rangle-\frac{eB_z}{m_ic}\frac{\langle n\rangle'}{\langle n\rangle}\langle\tilde v_x\tilde v_z\rangle-\frac{eB_z}{m_ic}\frac{\Gamma_n}{\langle n\rangle}\langle v_z\rangle'+\frac{eB_z}{m_ic}\frac{e}{m_i}\frac{\langle\tilde E_z\tilde n\rangle}{\langle n\rangle}\right]$$
$$=\int dx\left[-\frac{eB_z}{m_ic}\left(\langle\tilde v_z\mu_n\nabla_\perp^2\frac{\tilde n}{\langle n\rangle}\rangle-\nu\langle\tilde v_z\frac{\tilde n}{\langle n\rangle}\rangle+2\nu\langle\tilde v_z\rho_s^2\nabla_\perp^2\frac{e\tilde\phi}{T_e}\rangle\right)\right]$$





- Global balance: LHS = 0.0255 RHS = 0.0182

- Fluctuation helicity
$$\int dx\nu\langle\tilde{\mathbf v}\cdot\tilde\omega\rangle\sim\int dx\left[\frac{eB_z}{m_ic}\frac{\langle n\rangle'}{\langle n\rangle}\langle\tilde v_x\tilde v_z\rangle+\frac{eB_z}{m_ic}\frac{\Gamma_n}{\langle n\rangle}\langle v_z\rangle'\right]$$
particle flux
$$-\int dx\left[\frac{eB_z}{m_ic}\left(\langle\tilde v_z\mu_n\nabla_\perp^2\frac{\tilde n}{\langle n\rangle}\rangle-\nu\langle\tilde v_z\frac{\tilde n}{\langle n\rangle}\rangle\right)\right]$$
// sym. breaking required

Helicity of large scale flows

Fluctuation:

$$\partial_t \left[\langle \tilde{\mathbf{v}} \cdot \tilde{\omega} \rangle - 2\frac{eB_z}{m_i c} \left\langle \tilde{v}_z \frac{e\tilde{\phi}}{T_e} \right\rangle - 2\frac{c^2}{B_z^2} \left\langle \tilde{E}_y \partial_z \tilde{E}_x \right\rangle + \partial_x \langle \tilde{v}_y \tilde{v}_z \rangle \right]$$

$$+ \partial_x \left[-2\frac{eB_z}{m_i c} \langle \tilde{v}_x \tilde{q} \tilde{v}_z \rangle + 2\frac{eB_z}{m_i c}\frac{c^2}{B_z^2} \left\langle \tilde{E}_x \tilde{E}_z \right\rangle + 2\langle v_y \rangle' \langle \tilde{v}_x \tilde{v}_z \rangle \right]$$

Coupling to flows

$$+ 2\langle v_z \rangle' \partial_x \langle \tilde{v}_x \tilde{v}_y \rangle - 2\langle v_y \rangle' \partial_x \langle \tilde{v}_x \tilde{v}_z \rangle - 2\frac{eB_z}{m_i c}\frac{\langle n \rangle'}{\langle n \rangle} \langle \tilde{v}_x \tilde{v}_z \rangle - 2\frac{eB_z}{m_i c}\frac{\Gamma_n}{\langle n \rangle} \langle v_z \rangle'$$

$$= -2\frac{eB_z}{m_i c} \left(\langle \tilde{v}_z \mu_n \nabla_\perp^2 \frac{e\tilde{\phi}}{T_e} \rangle - \nu \langle \tilde{v}_z \frac{e\tilde{\phi}}{T_e} \rangle + 2\nu \langle \tilde{v}_z \rho_s^2 \nabla_\perp^2 \frac{e\tilde{\phi}}{T_e} \rangle \right)$$

Mean flow:

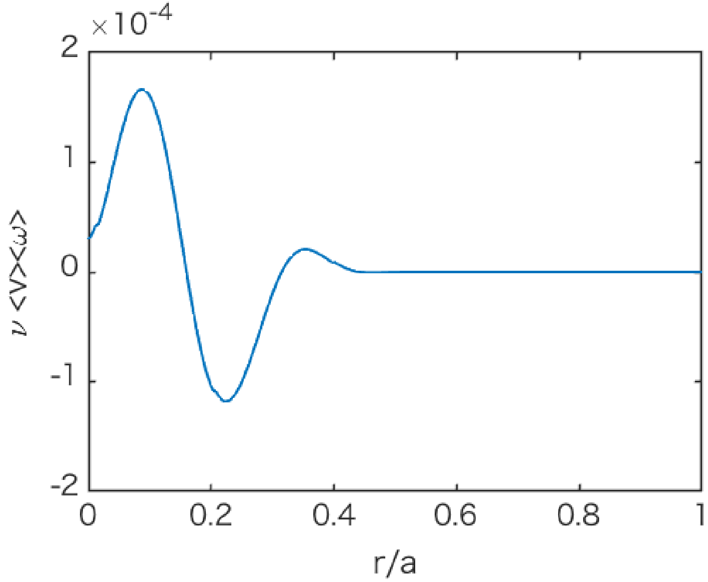
Forces

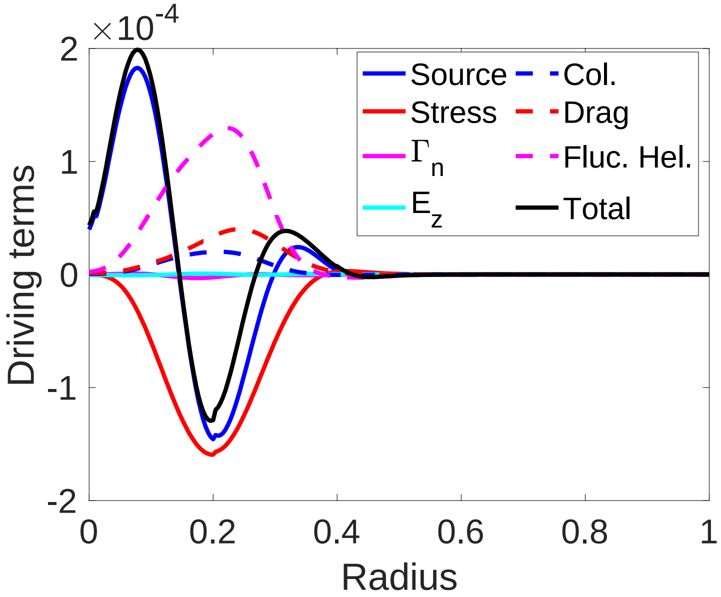
$$\int dx (\partial_t + \nu) \langle \mathbf{v} \rangle \cdot \langle \omega \rangle = \int dx \left[S_z \langle v_y \rangle' + \frac{eB_z}{m_i c} \left(\frac{\langle n \rangle'}{\langle n \rangle} \langle \tilde{v}_x \tilde{v}_z \rangle + \frac{\Gamma_n}{\langle n \rangle} \langle v_z \rangle' - \frac{e}{m_i} \frac{\langle \tilde{E}_z \tilde{n} \rangle}{\langle n \rangle} \right) \right]$$

$$+ \int dx \left[-\frac{eB_z}{m_i c} \left(\langle \tilde{v}_z \mu_n \nabla_\perp^2 \frac{\tilde{n}}{\langle n \rangle} \rangle - \nu \langle \tilde{v}_z \frac{\tilde{n}}{\langle n \rangle} \rangle + 2\nu \langle \tilde{v}_z \rho_s^2 \nabla_\perp^2 \frac{e\tilde{\phi}}{T_e} \rangle \right) \right]$$

Helicity of large scale flows

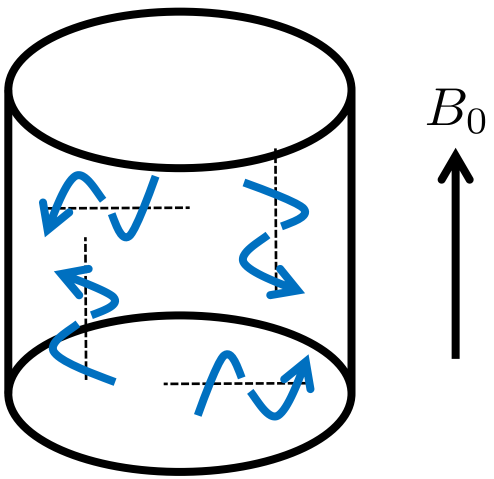
$$\int dx \nu \langle \mathbf{v} \rangle \cdot \langle \omega \rangle = \int dx \left[S_z \langle v_y \rangle' + \frac{eB_z}{m_i c} \left(\frac{\langle n \rangle'}{\langle n \rangle} \langle \tilde{v}_x \tilde{v}_z \rangle + \frac{\Gamma_n}{\langle n \rangle} \langle v_z \rangle' - \frac{e}{m_i} \frac{\langle \tilde{E}_z \tilde{n} \rangle}{\langle n \rangle} \right) \right]$$
$$+ \int dx \left[-\frac{eB_z}{m_i c} \left(\langle \tilde{v}_z \mu_n \nabla_\perp^2 \frac{\tilde{n}}{\langle n \rangle} \rangle - \nu \langle \tilde{v}_z \frac{\tilde{n}}{\langle n \rangle} \rangle + 2\nu \langle \tilde{v}_z \rho_s^2 \nabla_\perp^2 \frac{e\tilde{\phi}}{T_e} \rangle \right) \right]$$





- After integration: LHS = -0.84×10^{-4} RHS = -1.1×10^{-4}
- Mean helicity -> mainly due source + zonal flow shear

B-field generation by PVG turbulence

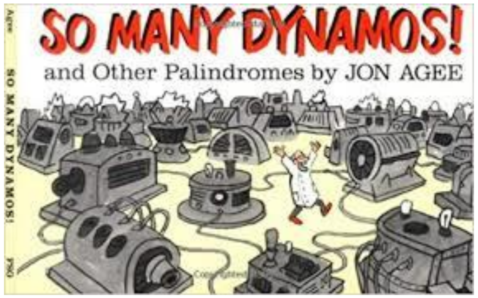


$$\mathbf{B} = B_0 \hat{z} + \tilde{\mathbf{B}}$$

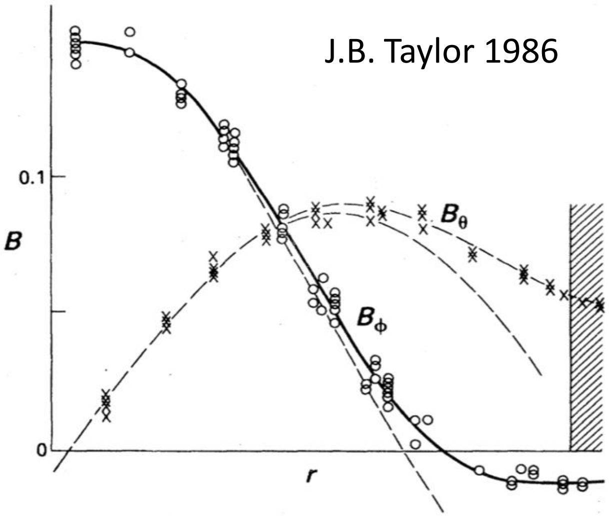
c.f. for tokamaks:

B_r -> density pump out

B_θ -> steady state operation



- For illustration, force free field: $\nabla \times \tilde{\mathbf{B}} = K\tilde{\mathbf{B}}$



J.B. Taylor 1986

$$B_z(r) \rightarrow J_\theta(r)$$

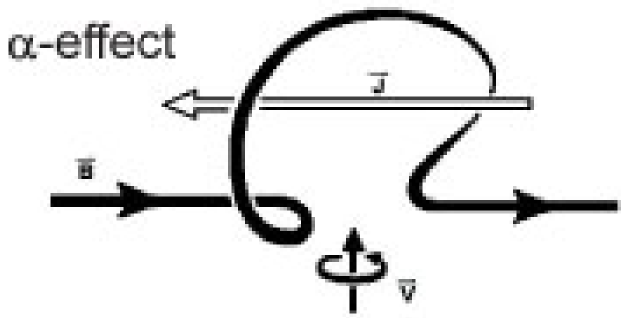
$$B_\theta(r) \rightarrow J_z(r)$$

✓ Force free condition can be satisfied

- ➢ B_0 modified
- ➢ B_θ self-sustained

B-field generation by PVG turbulence

- Amplification of force free field via alpha effect



$$\gamma = \alpha K - \eta K^2$$

amplification via helicity, α effect

Ohmic dissipation

- Condition for field amplification

$$\gamma = \alpha K - \eta K^2 > 0 \quad \Rightarrow \quad \frac{v_* a}{\eta} > aK \frac{\nu}{\omega_{ci}} \frac{L_n}{\tau} \frac{v_*}{\langle \tilde{v}_x \tilde{v}_z \rangle}$$

Magnetic Reynolds Number

Critical Momentum flux

Conclusion and Discussion

- Helicity of turbulent plasmas likely to be finite.
  - Demonstrated for DW + PVG. // symmetry breaking key.
  - Turbulent plasmas may be viewed as a soup of helices
- Turbulent plasmas may amplify large scale B-field.
  - In dynamo experiment, externally driven flows are a key. What of fluctuations?
- This year's theme: phase dynamics?
- Other applications? Flows in H-mode? Any other?

Helicity of turbulent plasmas could be a relevant parameter!