Physical Mechanisms at Play In the Multi-Scales Interaction Between MHD and Turbulence

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In collaboration with

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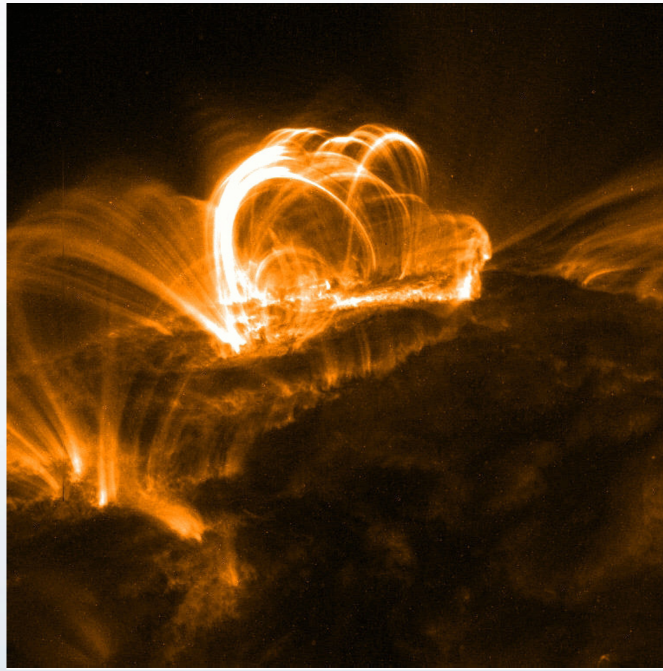
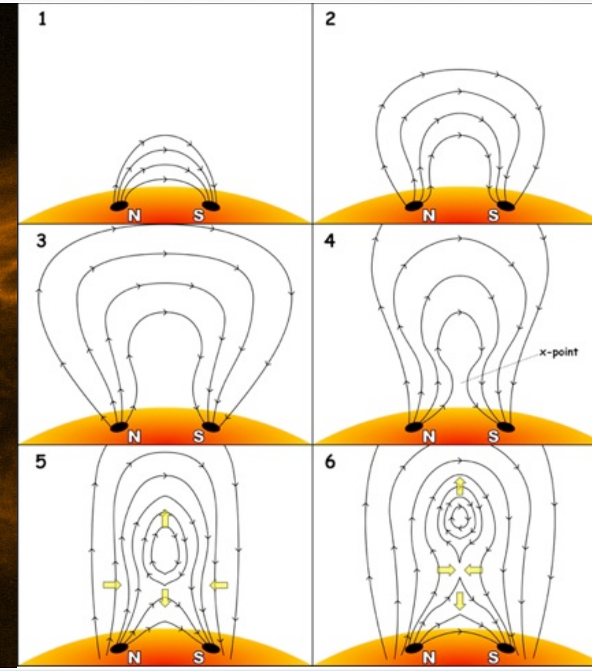
Magnetic islands are ubiquitous in nature

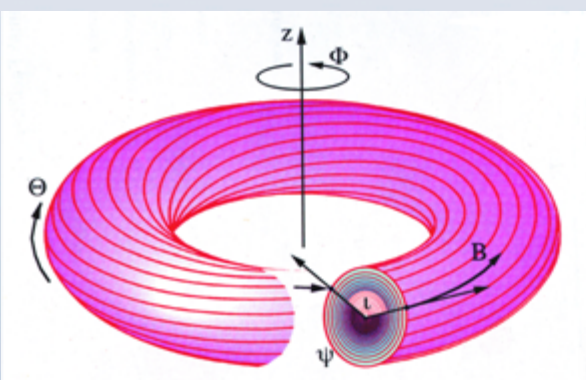
On Solar Surface

[September 2005, captured in the X-ray waveban by NASA's TRACE satellite. Photo courtesy of the University of California Berkeley, all rights reserved]

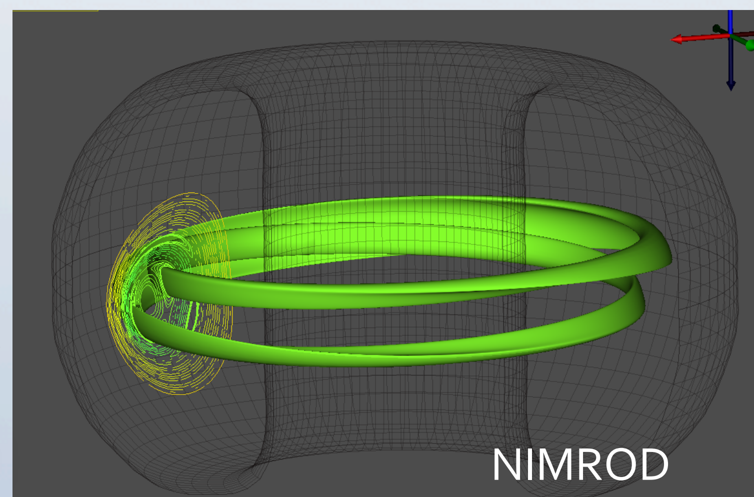




In tokamak



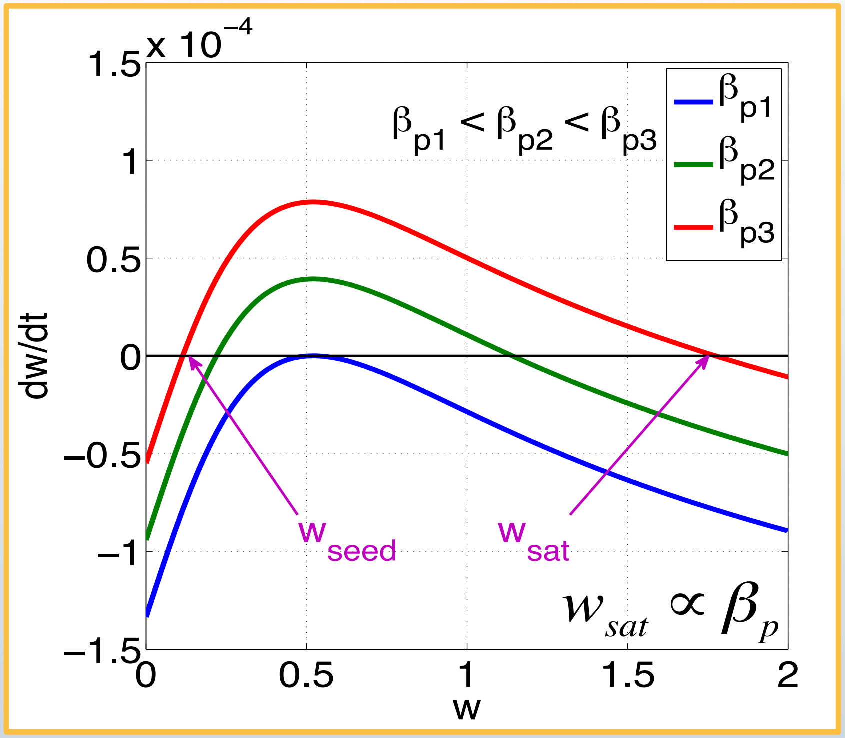
Current driven instability



Magnetic islands degrade tokamak confinement

- Growth of Neoclassical Tearing Modes (**NTMs**) : **magnetic island** degrading the plasma pressure and sometimes causing disruption

- Behavior of NTMs : modified **Rutherford equation**



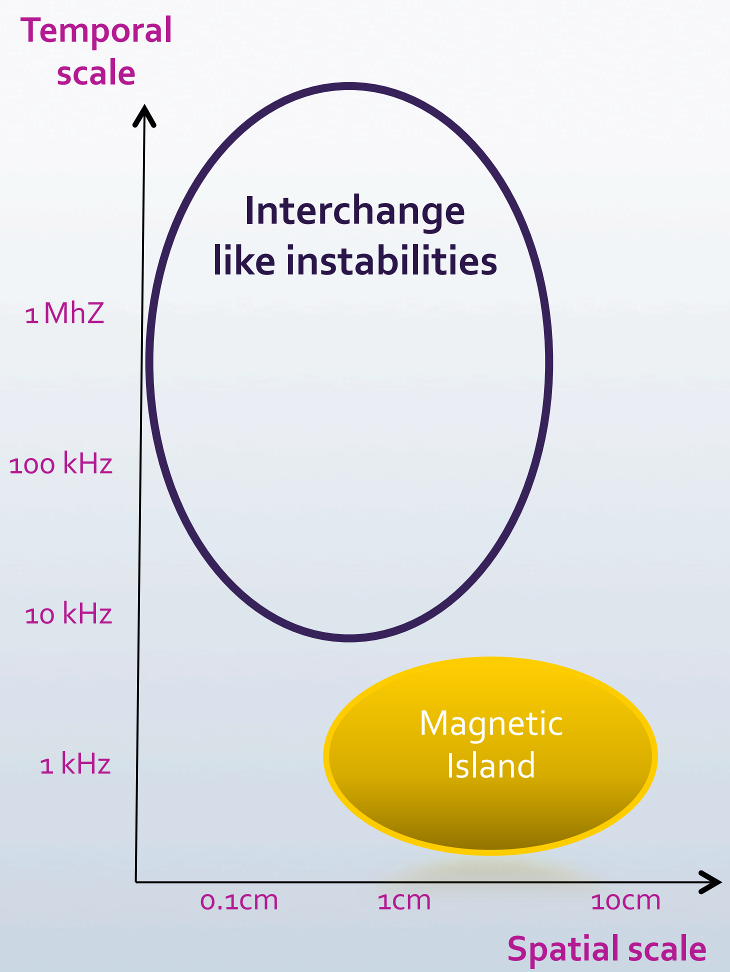
[R.J. La Haye, POP 13 (2006)]

- NTMs precursors :
  - ✧ Sawtooth oscillations
  - ✧ Fishbones instabilities
  - ✧ Edge localized modes
  - ✧ … ?

- In JT-60U, **80% of high β discharges**, a (2/1) NTM appears without precursor event **[A. Isayama et al, PFR 8 (2013)]**

Open question : origin of seed island ?

MHD-Turbulence Interaction, a Multi-Scales Problem



- Interchange like instabilities coexist with macro MHD instabilities and lead to micro-turbulence in fusion devices.

- The interaction of magnetic island with interchange is a multi-scales problem.

[F. Militello et al, POP 15 (2008)]
[F.L. Waelbroeck et al, PPCF 51 (2009)]
[M. Muraglia et al, PRL 103 (2009)]
[A. Ishizawa et al, POP 17 (2010)]
[F. Hariri et al, PPCF 57 (2015)]
[L. Bardoczi et al, POP 24 (2017)]

- **Turbulence Driven Magnetic Island (TDMI)**

[M. Muraglia et al, PRL 107 (2011)]
[A. Poyé et al, POP 22 (2015)]
[W. Hornsby et al, PPCF 58 (2015)]
[O. Agullo et al, POP 24 (2017)]
[A. Ishizawa et al, PPCF 61 (2019)]

- **TDMI amplified and at the origin of a NTM**

[M. Muraglia et al, NF (2017)]
J. Frank, PhD thesis, CEA and PIIM Lab

Outline

I. Introduction
MHD-Turbulence Interaction Modelisation

II. **Turbulence driven island** in a 2D slab geometry
(M. Muraglia)

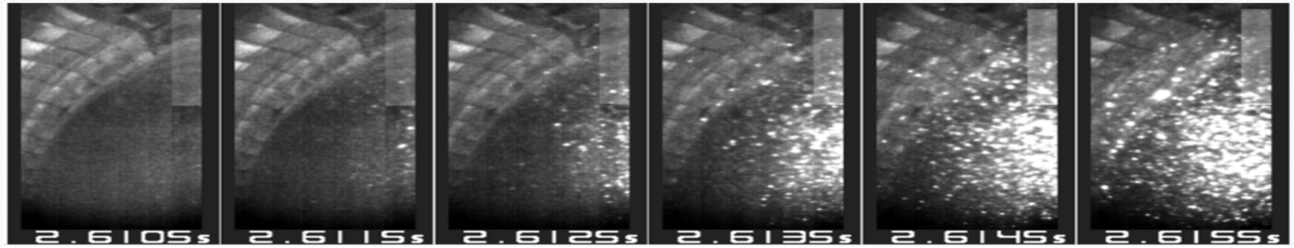
III. Nonlinear growth of **NTM from a seed turbulence driven island**
(M. Muraglia)

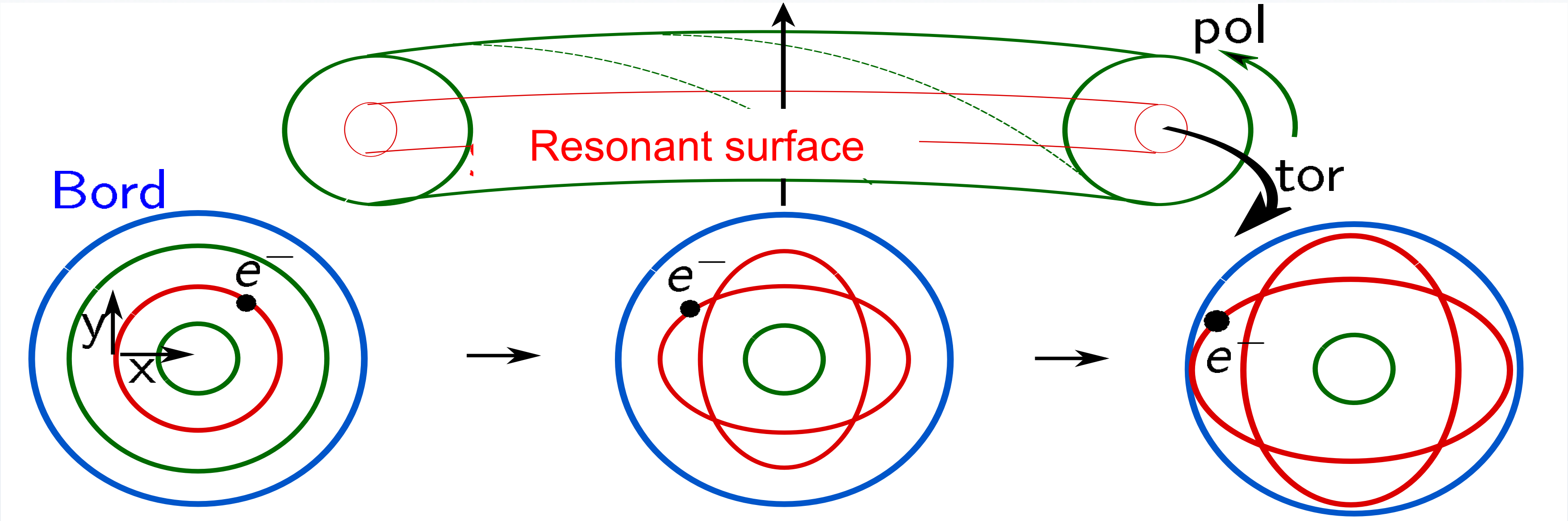
IV. Coherent and **non local beating** leading to island generation in a **3D geometry**
(N. Dubuit)

V. Conclusions

I. Magnetic island : Tearing Instability

Disruption :

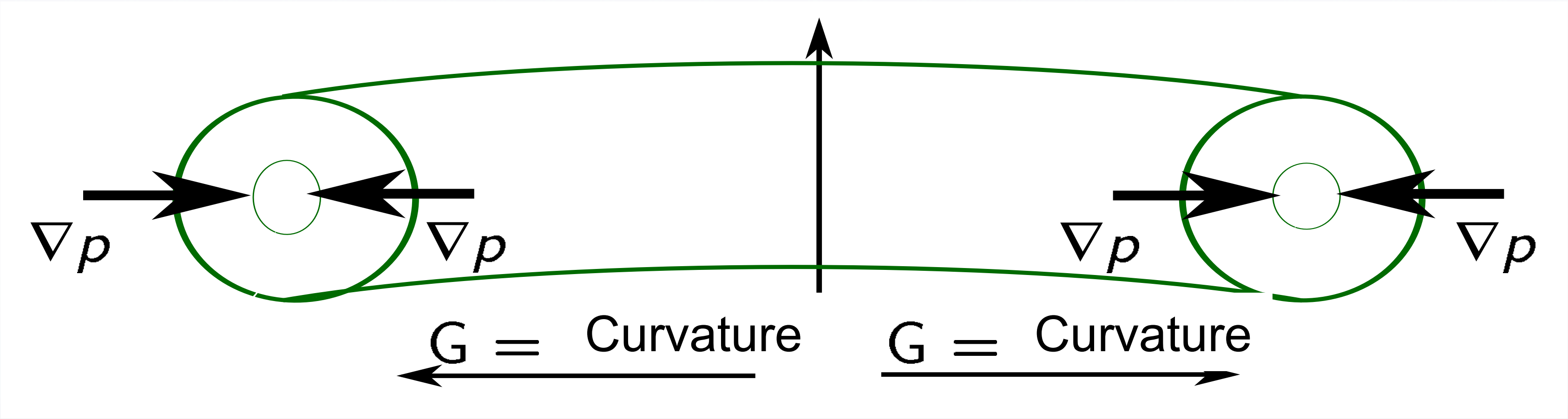




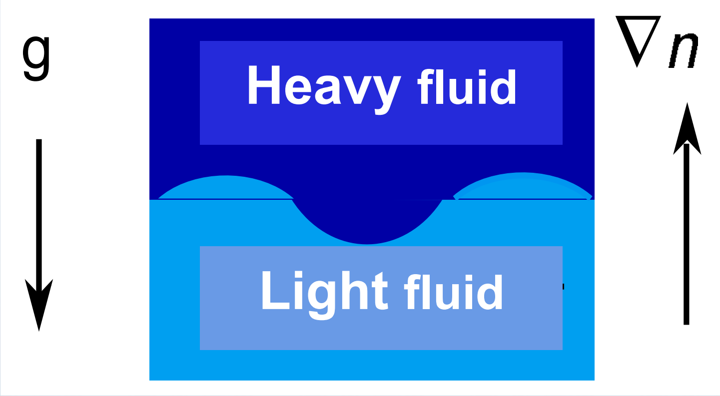
- Equilibrium
- Tearing instability
- Tearing and reconnection of magnetic field lines
- Generation of magnetic islands
- Disruption

Challenge : Control of magnetic island growth

I. Interchange Instability



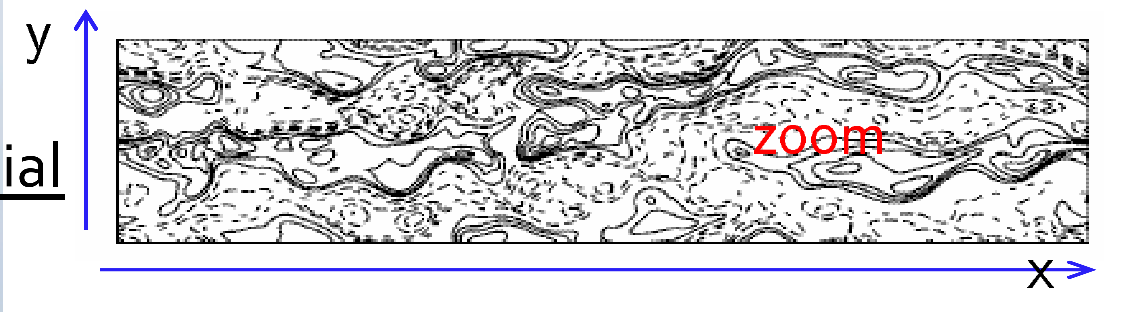
Analogy : Rayleigh-Taylor instability



Tokamak :

Curvature and pressure gradient in opposition
=> Interchange instability
=> Turbulence

Electrostatic potential fluctuations :



I. Model: Reduced MHD

- **Minimal Model :**
  - 2D Slab => near a resonant surface in a (x, y) poloidal cross section (mono-helicity)
  - Reduced MHD => Fluctuations dynamic evolution of electrostatic potential ϕ, electronic pressure *p* and magnetic flux ψ.

- Minimal **Interchange** Model :

Momentum conservation

$$\partial_t \nabla_\perp^2 \phi + [\phi, \nabla_\perp^2 \phi] = -\kappa_1 \partial_y P + \nu \nabla_\perp^4 \phi$$

Energy conservation

$$\partial_t P + [\phi, P] = -\partial_x P_0 \partial_y \phi + \chi_\perp \nabla_\perp^2 P$$

- Minimal **Tearing Mode** Model :

Momentum conservation

$$\partial_t \nabla_\perp^2 \phi + [\phi, \nabla_\perp^2 \phi] = [\psi + \psi_0, \nabla_\perp^2 \psi] + \nu \nabla_\perp^4 \phi$$

Ohm's law

$$\partial_t \psi = [\psi + \psi_0, \phi] + \eta \nabla_\perp^2 \psi$$

- Model includes both resistive **Interchange** and **Tearing Mode** :

$$\partial_t \nabla_\perp^2 \phi + [\phi, \nabla_\perp^2 \phi] = [\psi + \psi_0, \nabla_\perp^2 \psi] - \kappa_1 \partial_y P + \nu \nabla_\perp^4 \phi$$

$$\partial_t P + [\phi, P] = -\partial_x P_0 ((1 - \kappa_2) \partial_y \phi + \kappa_2 \partial_y P) + \rho_\star^2 [\psi + \psi_0, \nabla_\perp^2 \psi] + \chi_\perp \nabla_\perp^2 P$$

$$\partial_t \psi = [\psi + \psi_0, \phi - P] - \partial_x P_0 \partial_y \psi + \eta \nabla_\perp^2 \psi$$

[M. Muraglia et al, NF 49, 055016 (2009)]

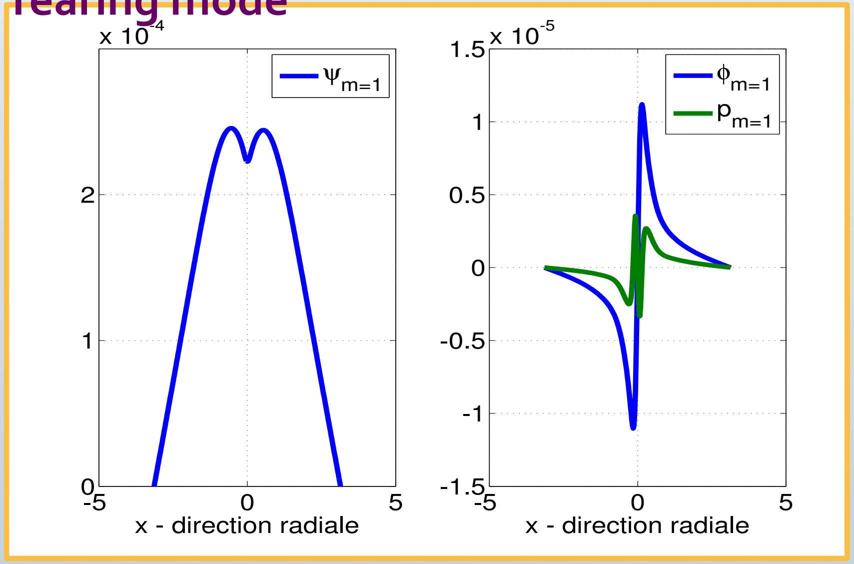
I. Model: Reduced MHD

- Numerical resolution of the model => Code AMON
  - temporal evolution : Runge-Kutta order 4th
  - semi-spectral code in a numerical box [Lx, Ly]
    finite difference in the radial x direction
    Fourrier decomposition in the poloidal y direction

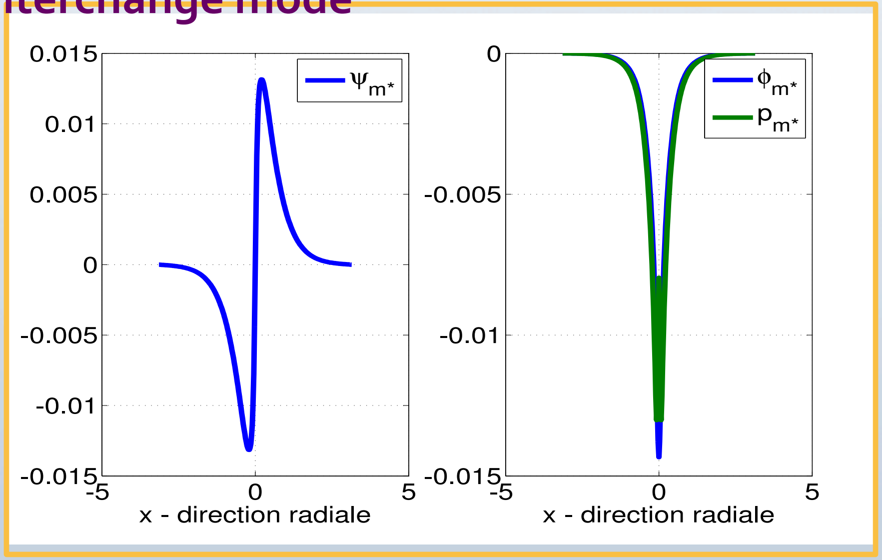
$$\psi\,(x,\,y,\,t) = \sum_{m\in\mathbb{Z}} \psi_m\,(x,\,t)\,exp^{i\,\frac{2\pi m}{Ly}\,y}$$

- Instability characterization : **THE PARITY** of the eigenfunctions $\psi_m,\ \phi_m,\ P_m$

Tearing mode

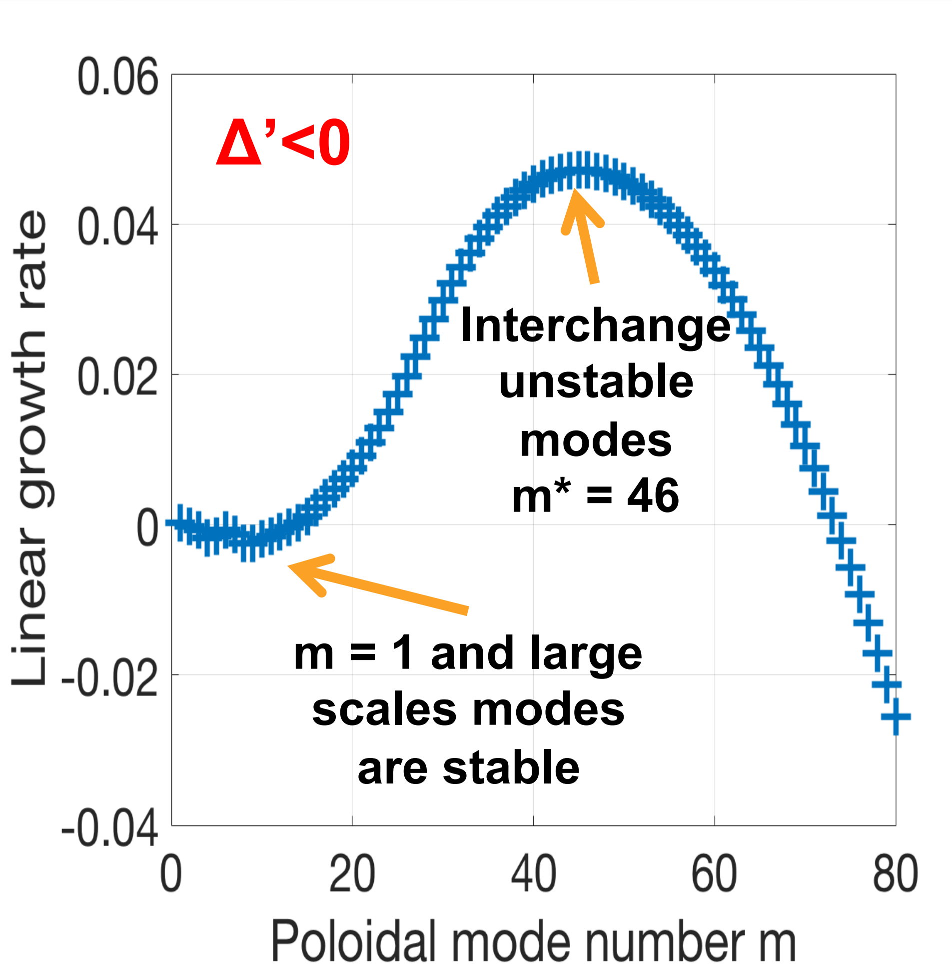


Interchange mode



II. NL generation of TDMI

Linear Spectrum



- Δ’<0 Linear spectrum is stable with respect with tearing instability

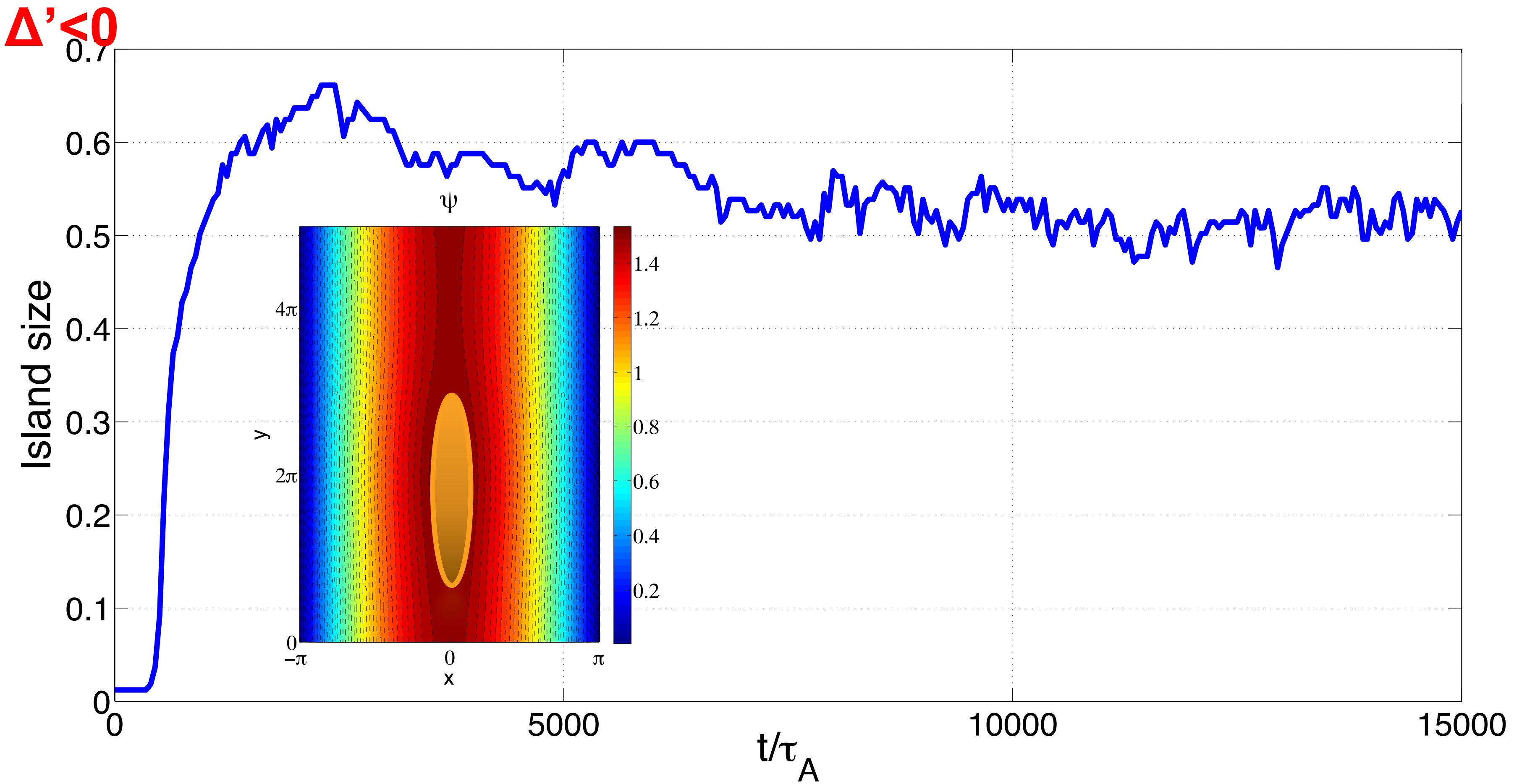
  => **No island**

- Stable large scales modes

- Small scales turbulence driven by interchange instability

=> **What ‘s about non-linear dynamics ?**

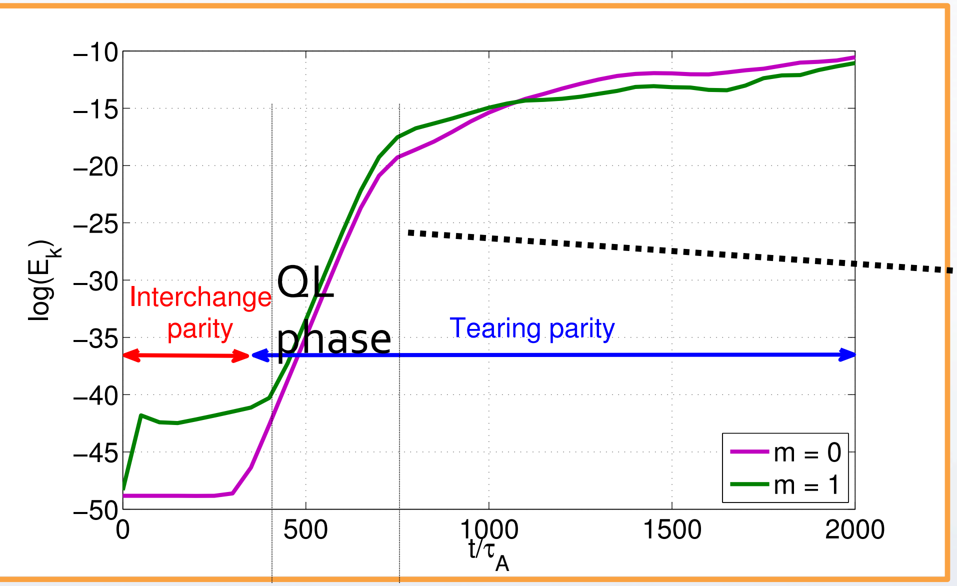
II. NL generation of TDMI

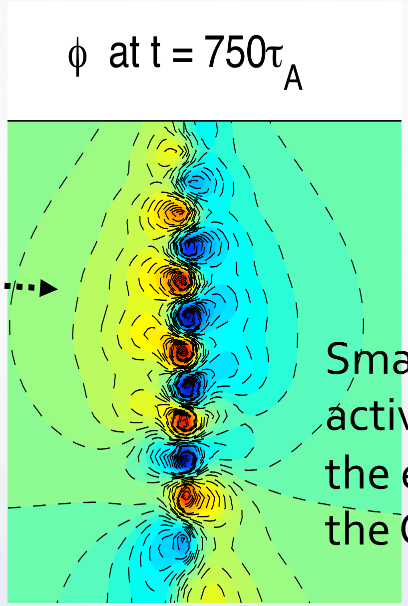


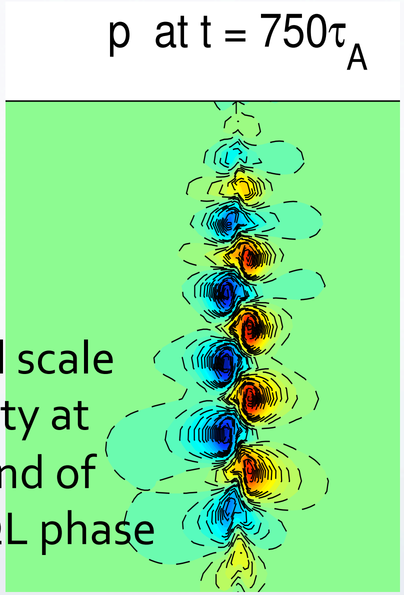
◆ NL generation of TDMI by a beating of interchange modes

[M. Muraglia et al, PRL 107 (2011)] & [A. Poyé et al, POP 22 (2015)]
[W. Hornsby et al, PPCF 58 (2015)]

II. Origine of the island





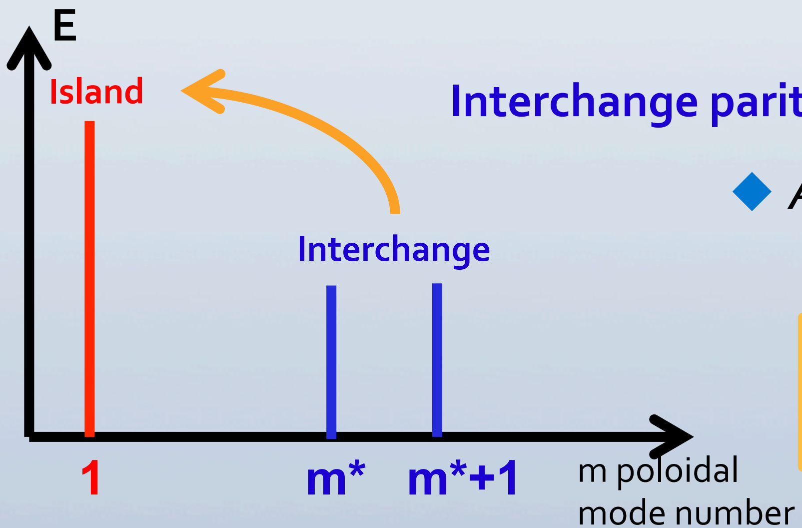


Small scale activity at the end of the QL phase

- Ohm's law projection on the mode m=1 :

$$\partial_t \psi_1 = (\partial_x \psi_{m\star})\, \phi_{m\star+1} exp^{i\frac{2\pi}{Ly}} + \ \ldots$$

Interchange parity : Odd Even

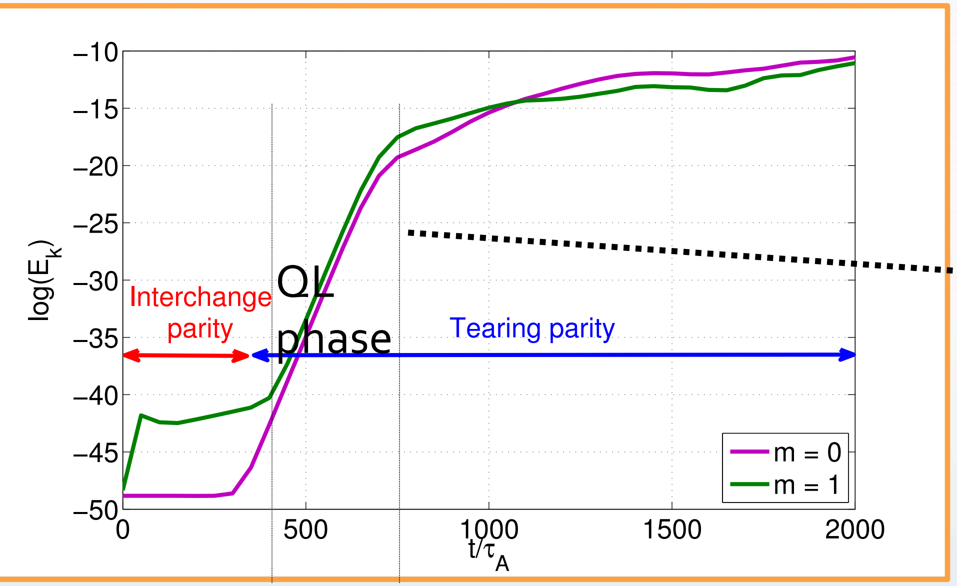


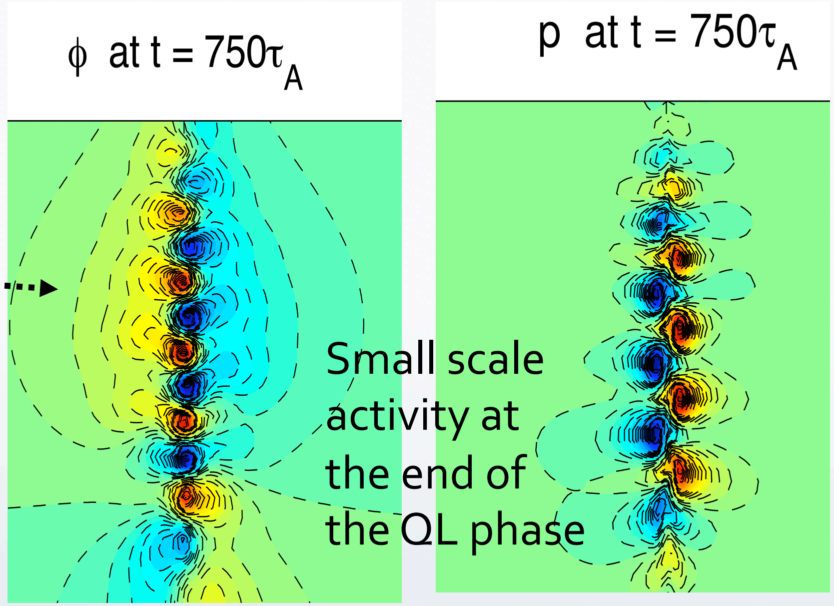
- *All the non linearities* of the model satisfy :

$$[\mathrm{Int}_{ss}, \mathrm{Int}_{ss}] \rightarrow \mathrm{Tear}_{ls}$$

Magnetic island generation by nonlinear beating of interchange modes

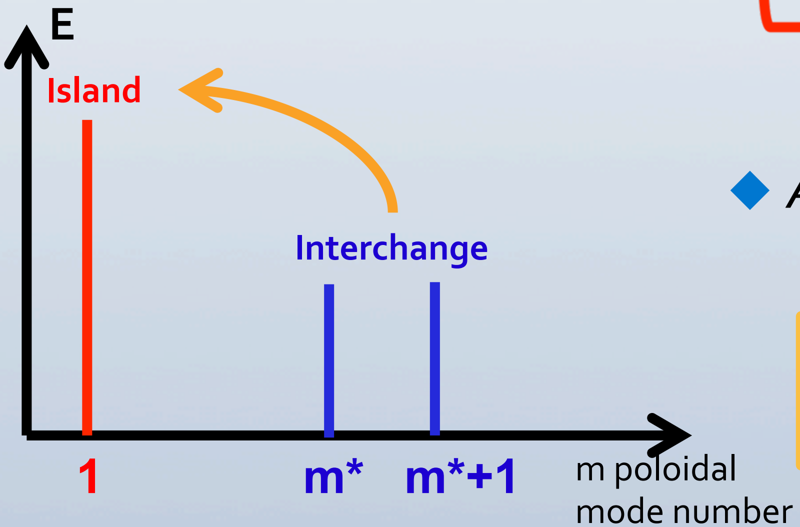
II. Origine of the island





- Ohm’s law projection on the mode m=1 :

$$\partial_t \psi_1 = \underbrace{(\partial_x \psi_{m\star})\, \phi_{m\star+1} exp^{i\frac{2\pi}{Ly}}}_{\text{Even => Tearing Parity}} + \; ...$$



- *All the non linearities* of the model satisfy :

$$[\mathsf{Int}_{ss}, \mathsf{Int}_{ss}] \rightarrow \mathsf{Tear}_{ls}$$

Magnetic island generation by nonlinear beating of interchange modes

III. NL growth of NTM ?

- Question :

Is such turbulence driven seed island at the origin of NTM growth ?

=> New model including bootstrap current is required

- In 2D slab geometry :

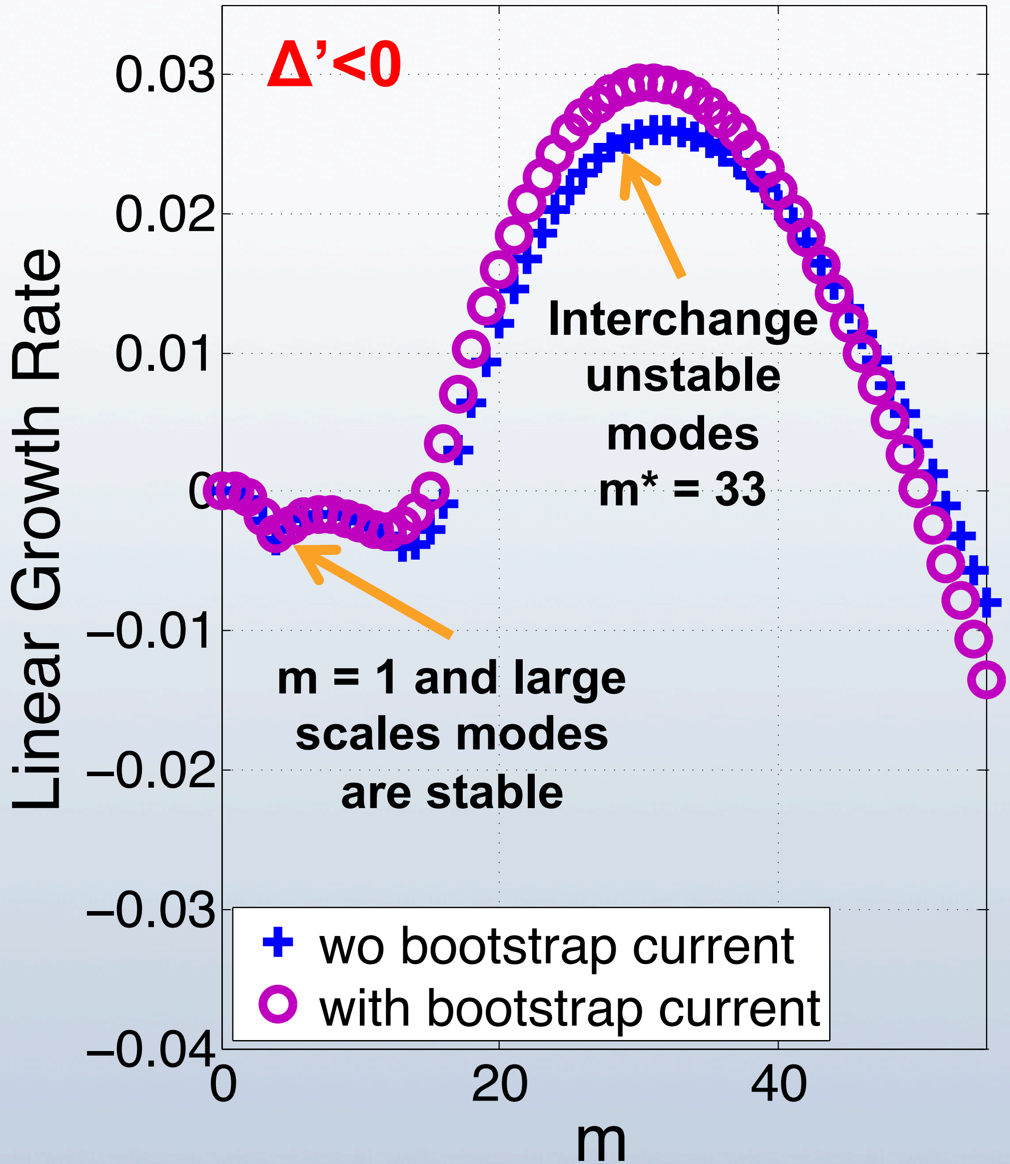
$$\partial_t \nabla_\perp^2 \phi + [\phi, \nabla_\perp^2 \phi] = [\psi + \psi_0, \nabla_\perp^2 \psi] - \kappa_1 \partial_y P + \nu \nabla_\perp^4 \phi$$

$$\partial_t P + [\phi, P] = -\partial_x P_0((1-\kappa_2)\partial_y \phi + \kappa_2 \partial_y P) + \rho_\star^2 [\psi + \psi_0, \nabla_\perp^2 \psi] + \chi_\perp \nabla_\perp^2 P$$

$$\partial_t \psi = [\psi + \psi_0, \phi - P] - \partial_x P_0 \partial_y \psi + \eta \nabla_\perp^2 \psi + \eta C_b \partial_x P$$

III. NL growth of NTM ?

Linear Spectrum



- Δ'<0 Linear spectrum is stable with respect with tearing instability

  => **No island**

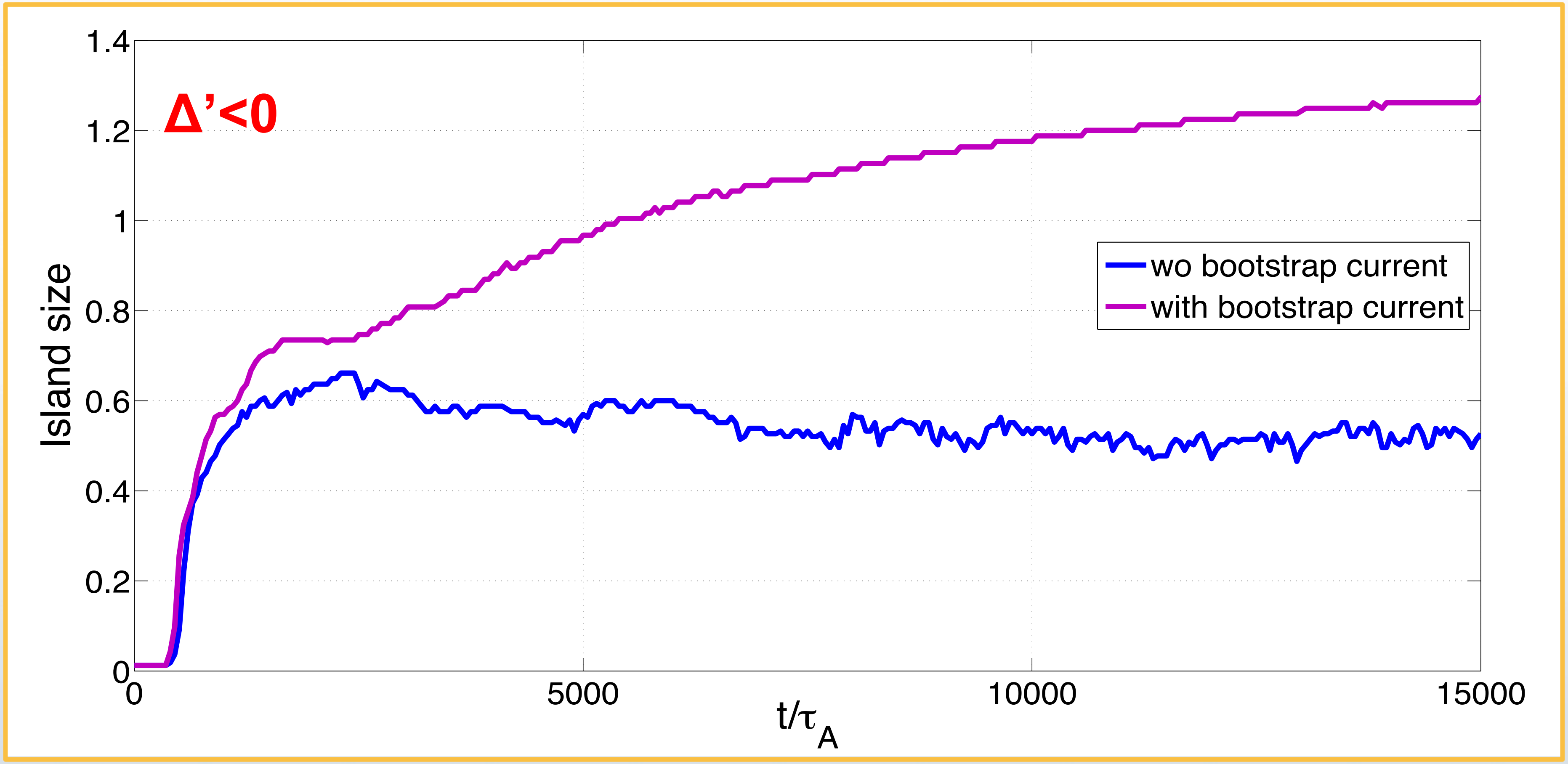
- Stable large scales modes

- Small scales turbulence driven by interchange instability

- Bootstrap current has a week effect on the linear spectrum

=> What 's about non-linear dynamics ?

III. NL amplification of TDMI by bootstrap current



◆ **Self-consistent generation of NTM from TDMI**

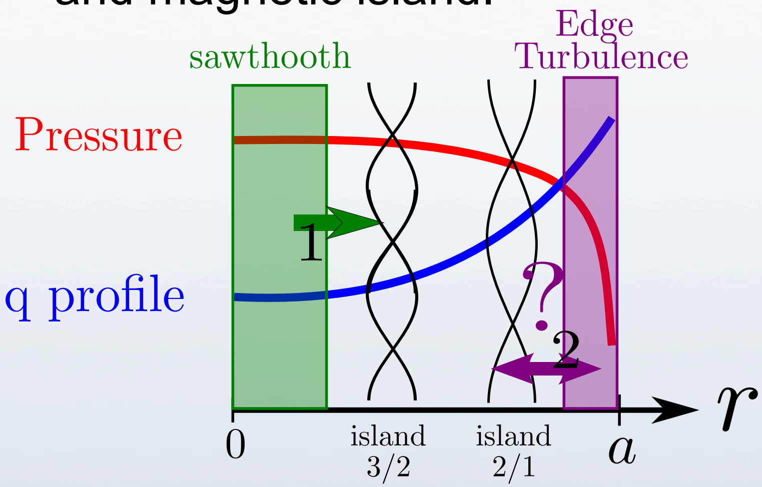
1. TDMI formation => **Seeding regime**
2. NL growth of NTM => **Amplification (by bootstrap current) regime**

[M. Muraglia et al, NF (2017)]

IV. Model in 3D cylindrical geometry

◆ In 2D slab geometry, turbulence and magnetic island are located around the same resonant surface => Islands everywhere in tokamaks ?

◆ In fusion devices, no systematic overlap between turbulence area and magnetic island.



1 M.F.F Nave, et al. Nuc. Fus. 43 (2003)
2 A. Isayama, et al. Plasma and Fusion Research 8 (2013)

Question : Nonlinear beating of interchange mode is still efficient ?

◆ 3D cylindrical Reduced MHD model :

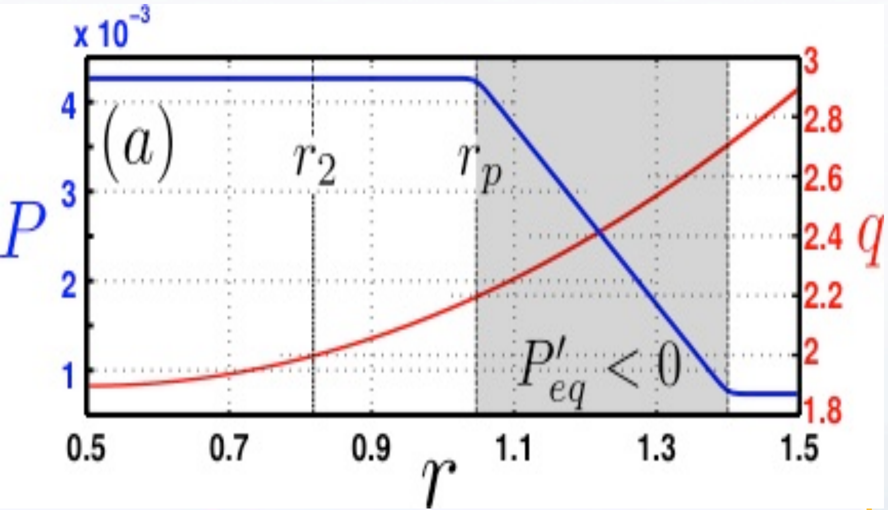
$$\partial_t \tilde{\psi} = \nabla_{\parallel} \tilde{\phi} - \nabla_{\parallel} (P_{eq} + \tilde{p}) + \eta \tilde{j}$$

$$\partial_t \tilde{\omega} + \{\tilde{\phi}, \tilde{\omega}\} = \nabla_{\parallel} (J_{eq} + \tilde{j}) - \frac{\kappa_1}{r} \partial_\theta \tilde{p} + \nu \triangle_{\perp} \tilde{\omega}$$

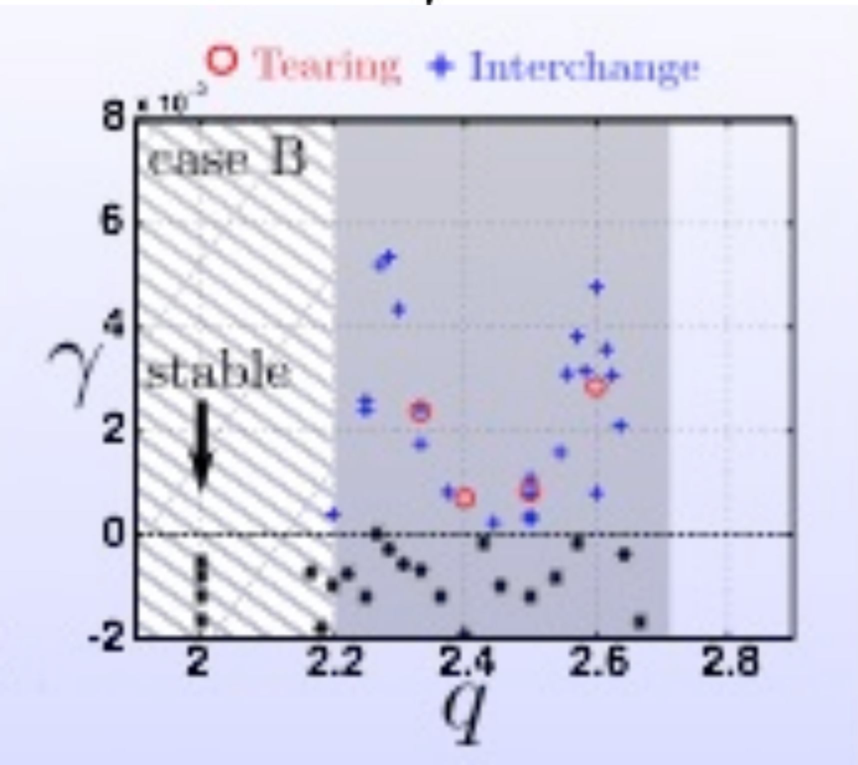
$$\partial_t \tilde{p} = \{P_{eq} + \tilde{p}, \tilde{\phi}\} + \rho^{*2} \{\Psi_{eq} + \tilde{\psi}, J_{eq} + \tilde{j}\} + \chi_{\perp} \triangle_{\perp} \tilde{p}$$

A simple model including interchange and Tearing instabilities.

IV. Numerical set-up and Linear stage



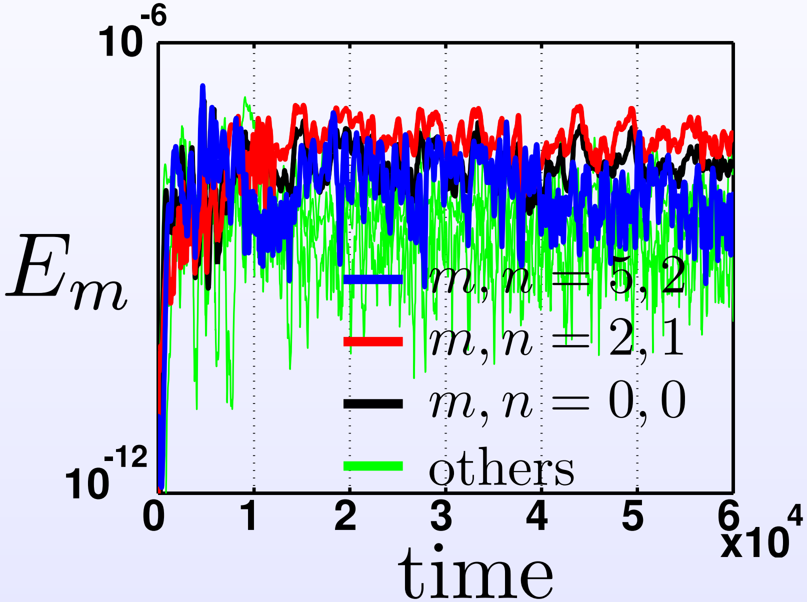
- Edge turbulence level is controlled through pressure equilibrium gradient.
- The q=2 surface is not located in an interchange area.
- Parameters choosen such as q=2 surface is stable with respect to the tearing instability.

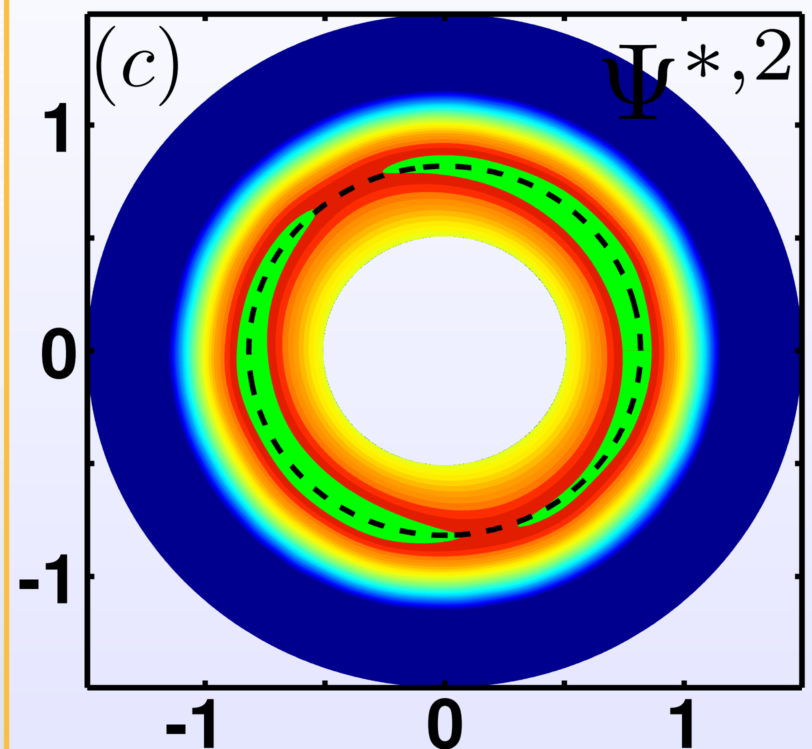


- No tearing mode develops at q=2.
- Interchange modes develop at the edge.

<u>Question</u> : Can we get a NL « spreading » of turbulence beating and generation of magnetic island at q=2 surface ?

IV. Nonlinear generation of q=2 island





◆ The dominant mode, in nonlinear phase, is located at the lowest rational surface in the whole box : q=2. It is in a stable zone (with restect to interchange and tearing instability) and produces a (2, 1) magnetic island.

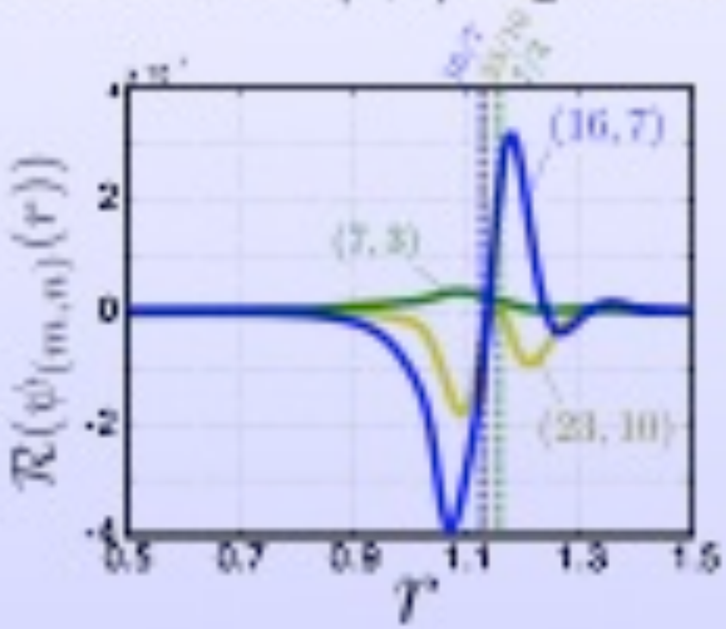
Question : What is the island generation mechanisms ?

IV. Beating of dominant unstable modes

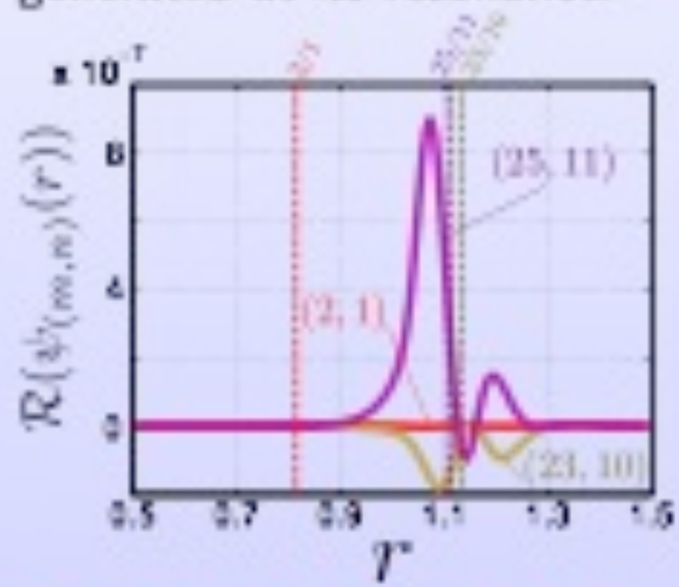
◆ <u>Nonlinear beating rules</u> :

1. The modes beat if they overlap.
2. The beating is effcient if the resulting mode is resonant at its birth location.

$(23,10)+(16,7)=(7,3)$. Rules are satisfied, the $(7,3)$ is generated.



$(25,11)+(23,10)=(2,1)$. Rules are not satified, the $(2,1)$ is not generated at its resonance.



Then, how to explain the growth of the mode (2, 1) ?

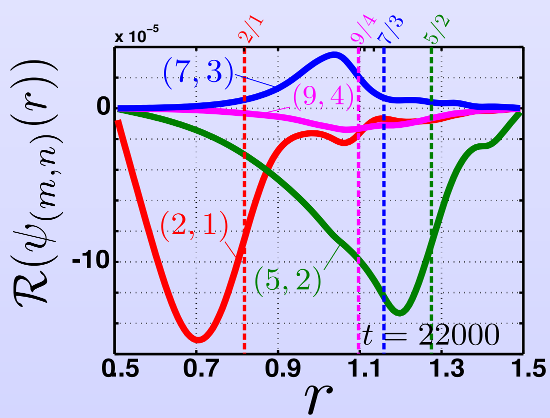
IV. Coeherent and non local beating

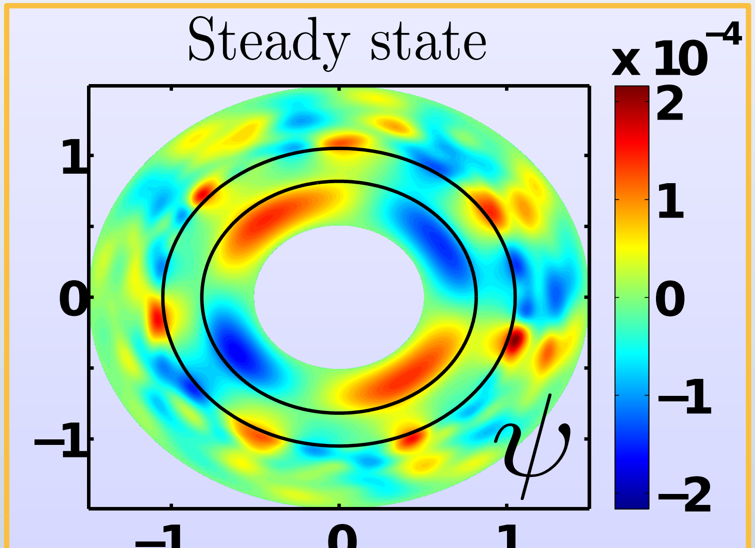
- The beating mechanisms produces modes with large radial structure (5, 2), (7, 3) and (9, 4) in the QL phase and remains efficient in the whole NL phase.

- The beating of such modes generates (2, 1) but only at the tail of the eigen function, at q=2.

$$(9,4)+(7,3)=(2,1)$$

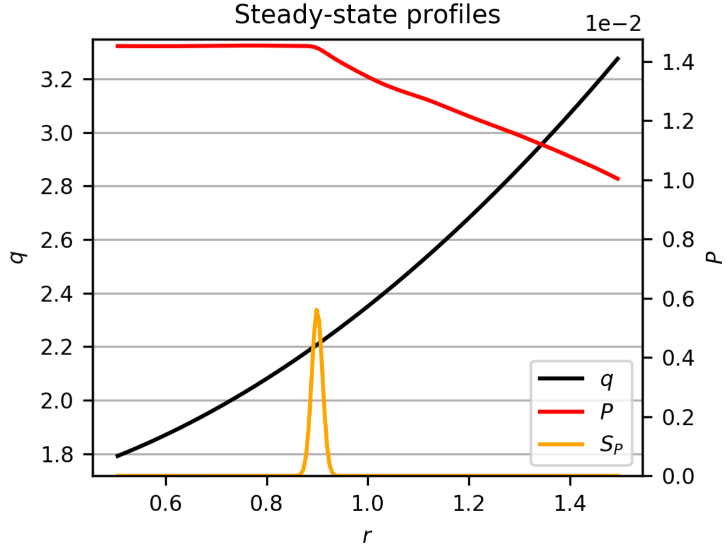
$$(7,3)+(5,2)=(2,1)$$

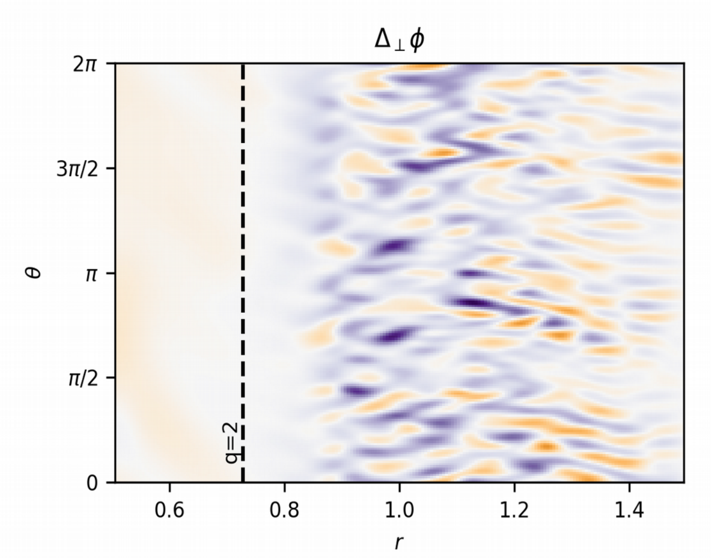




[A. Poyé et al, POP 22 (2015)]

IV. Toroidal source-driven model





Simple flux-driven model for toroidal interchange

$$\partial_t \omega + [\phi, \omega] = -\kappa_1 \mathcal{G} P + \nabla_{\|} J + \nu \Delta_\perp \omega$$

$$\partial_t P + [\phi, P] = \kappa_2 \mathcal{G} \phi + \kappa_2^P \mathcal{G} P + \rho_\star^2 [\psi, J] + \chi \Delta_\perp P + S_P$$

$$\partial_t \psi = \nabla_{\|}(\phi - P) + \eta j_{\|}$$

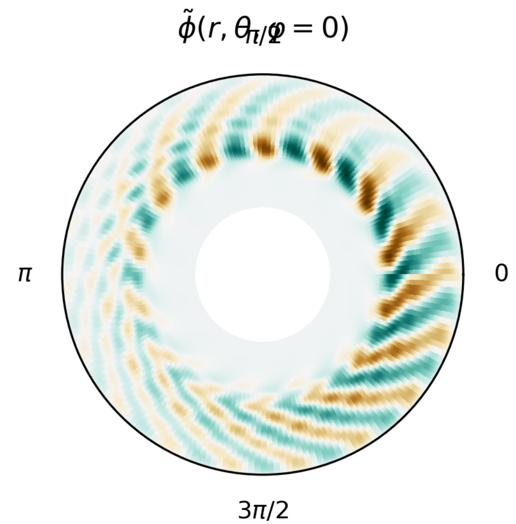
where

$$\mathcal{G} = \sin\theta \partial_r + \frac{1}{r} \cos\theta \partial_\theta$$

is the curvature operator.

Source term :
- constant and controlled profile throughout saturation phase
- allows consistent dynamics close to marginal stabiltiy

IV. Multiple coupling possibilities

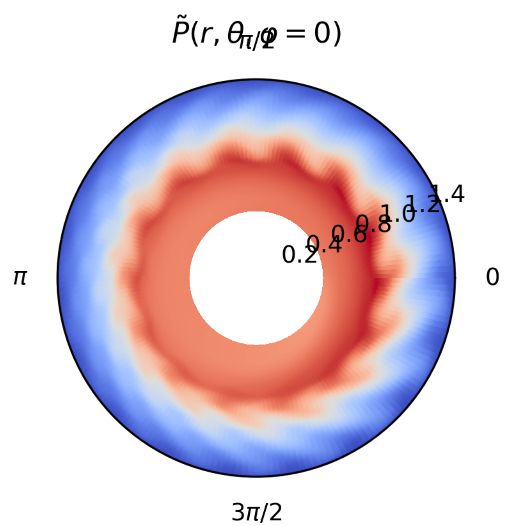


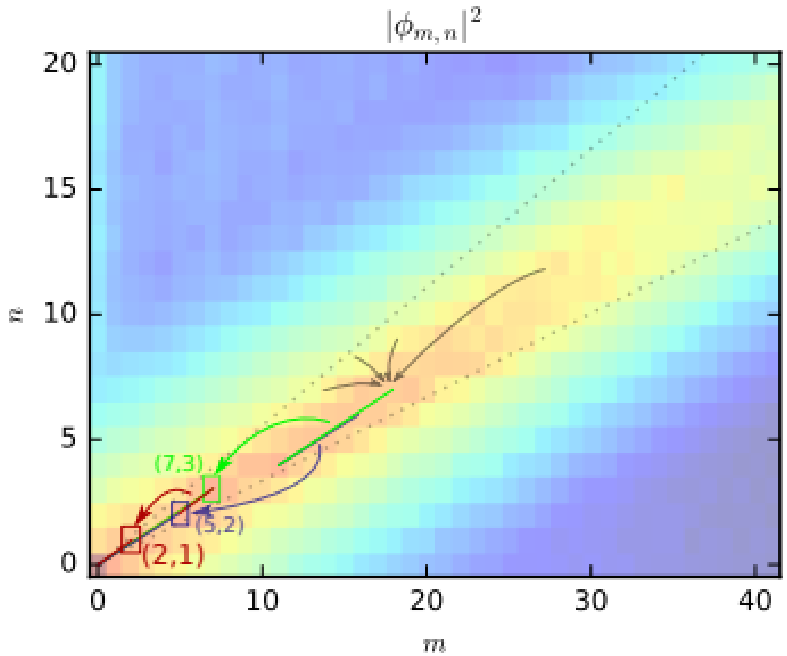
$$\mathcal{G} f = \sin\theta\,\partial_r f + \frac{1}{r}\cos\theta\,\partial_\theta f$$

Additional m ↔ m±1 coupling

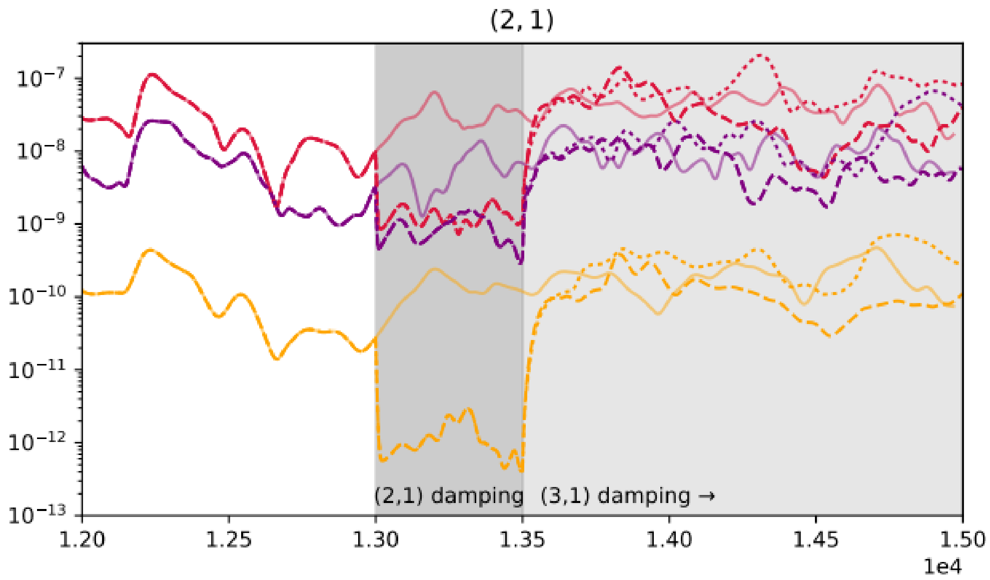
Mode couplings :

- Local nonlinear (conserves parity)
- Nonlocal nonlinear
- Linear toroidal





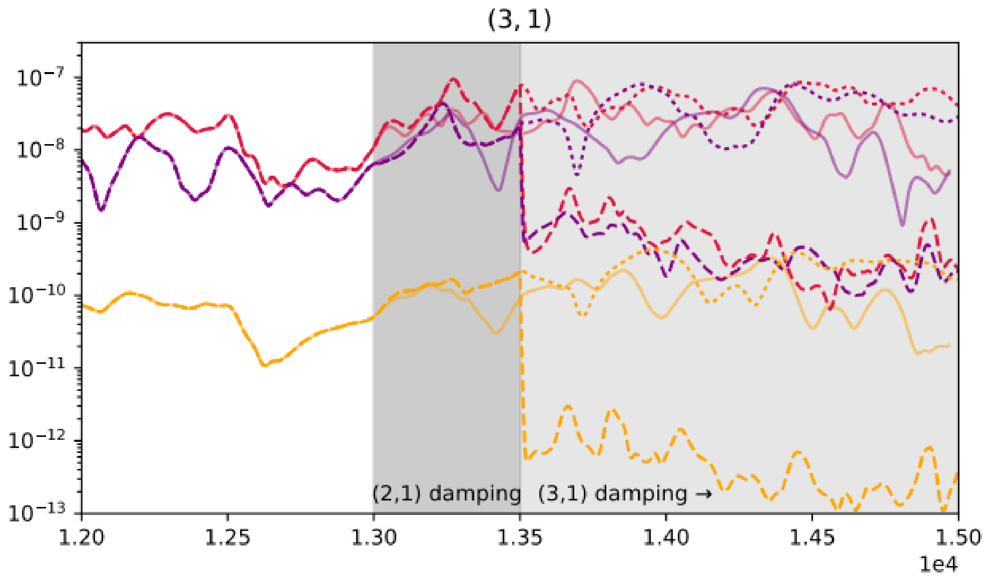
IV. Toroidal coupling



Direct coupling ?

(3,1) in the turbulent zone

(2,1) in the stable zone ?



Kill artificially (2,1) mode
Let it recover

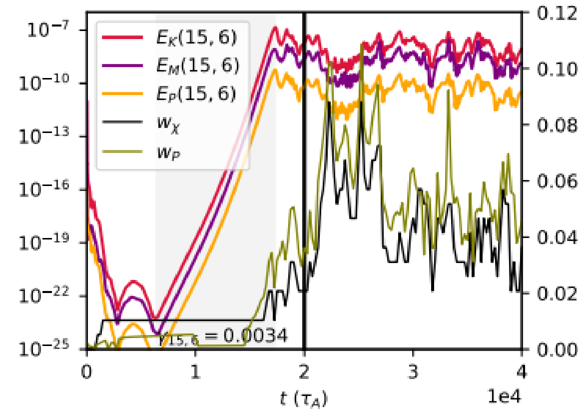
- Suppressing (3,1) mode
- Leaving (3,1) mode

Same recovery in both cases

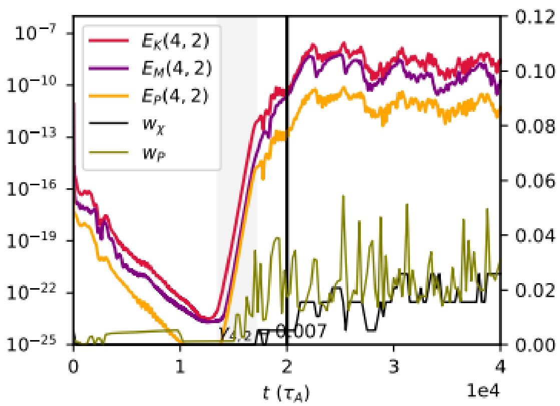
Role of (3,1) mode negligible

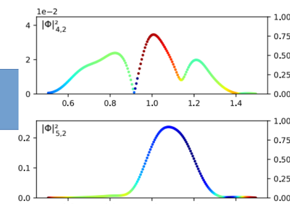
IV. Toroidal coupling

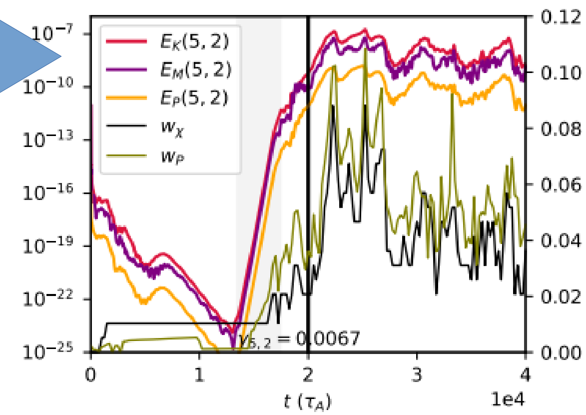
Low-source case :
new dynamics

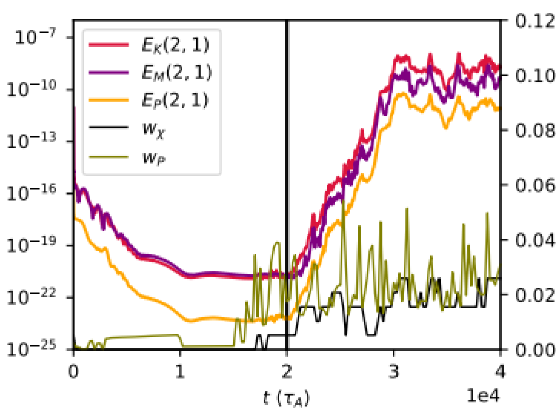


Quiescent zone









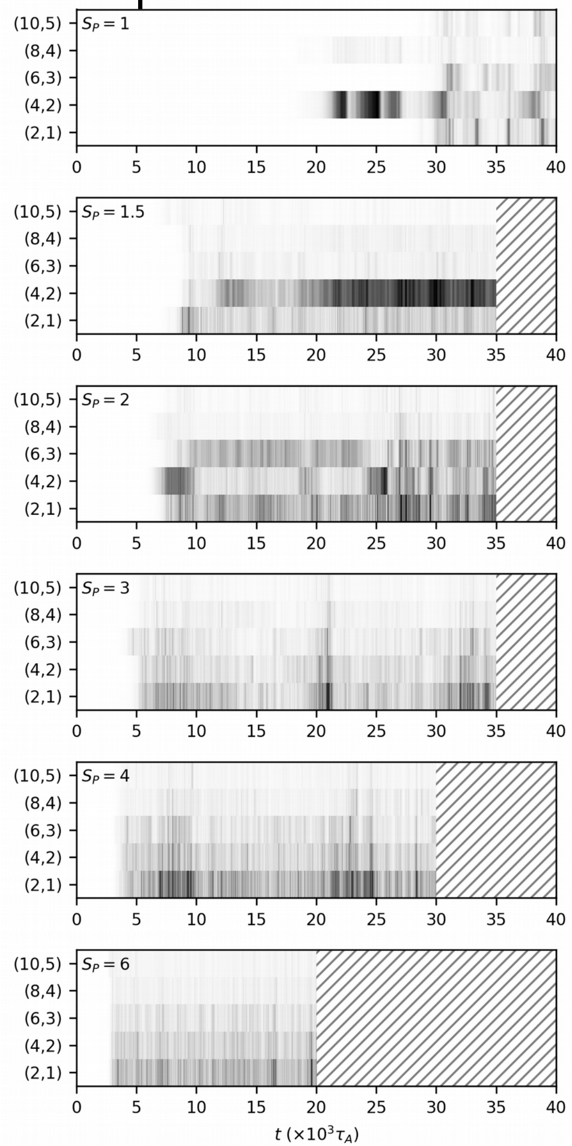
Turbulent zone

High-source: medium-scale mode beating dominant
(5,2) + (7,3) → (2,1)

≠ turbulence spreading

IV. Spectrum dynamics

q=2 resonant modes



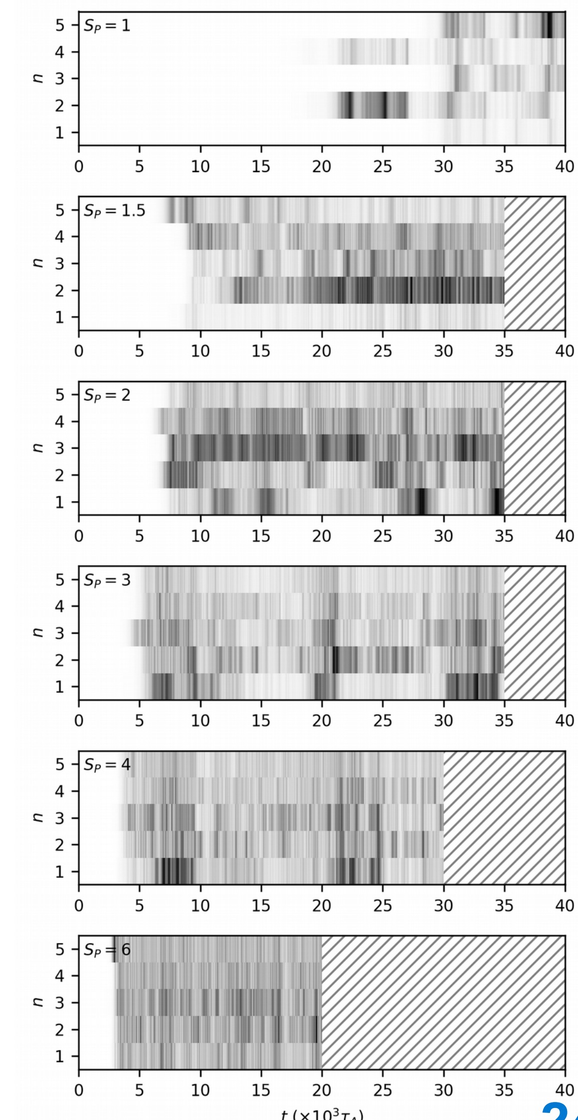
Increasing power source (feeding more energy to turbulence)

Remote island spectrum evolves towards small mode numbers

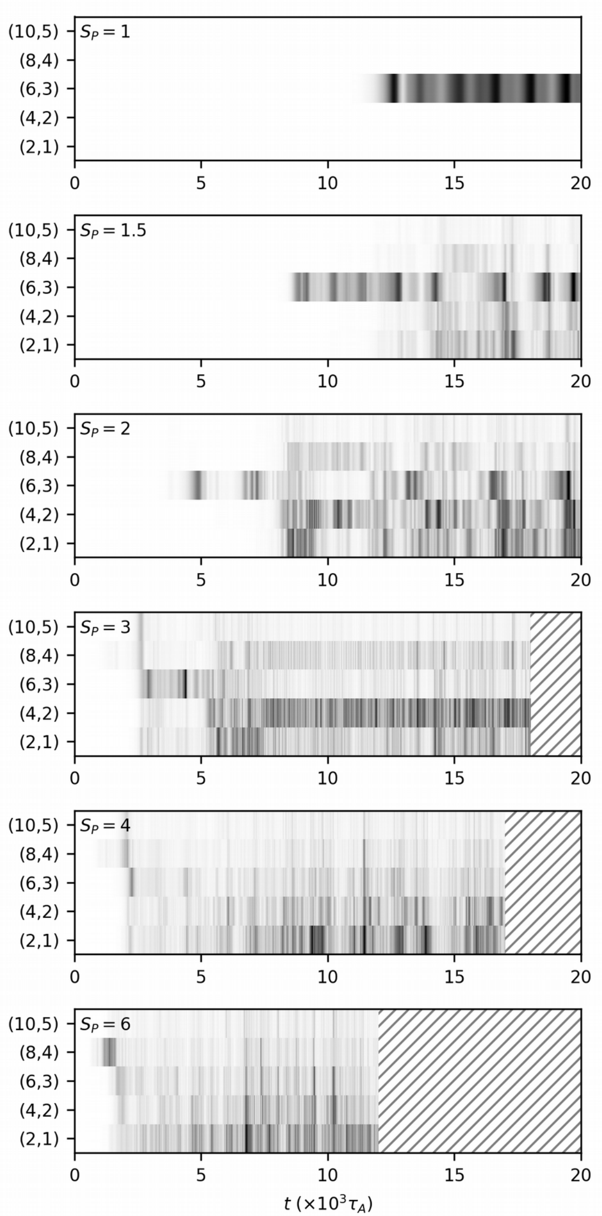
Direct toroidal coupling of medium-scale modes

Nonlinear coupling of medium scale modes

all modes



IV. Spectrum dynamics



Special case : tearing-like modes slightly unstable

→ Dominated by medium-scale tearing (5,2), (7,3)

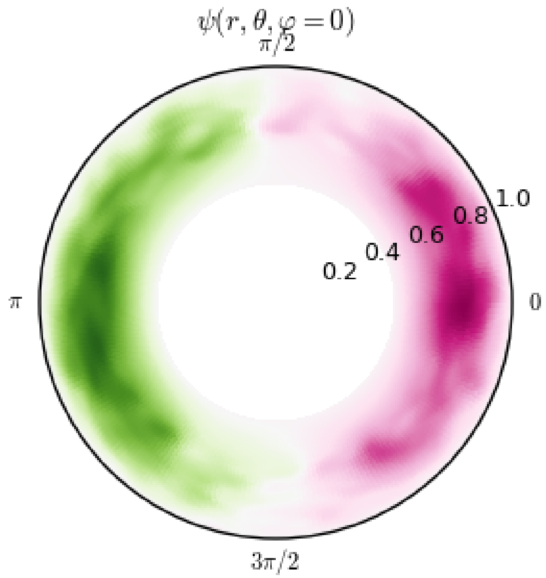
Final state goes from n=3 to n=1 with increasing source

Intermediate final values → oscillating/chaotic regimes

Integer final values → steady-state regime
(with lower maximum island size)

Also : generation mechanism *independent on underlying instability*

IV. Island size and structure



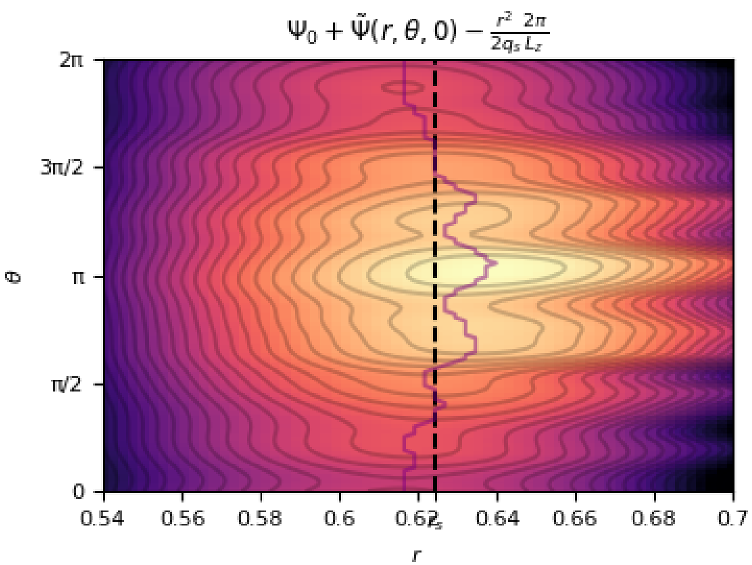
Circular concentric flux surfaces
are not a toroidal equilibrium state

$$\partial_t \omega + [\phi, \omega] = -\kappa_1 \mathcal{G} P + \nabla_{\|} J + \nu \Delta_{\perp} \omega$$

$$\partial_t P + [\phi, P] = \kappa_2 \mathcal{G} \phi + \kappa_2^P \mathcal{G} P + \rho_\star^2 [\psi, J] + \chi \Delta_{\perp} P + S_P$$

$$\partial_t \psi = \nabla_{\|} (\phi - P) + \eta j_{\|}$$

→ Generation of an axisymmetric perturbation of ψ and Φ from pressure gradient

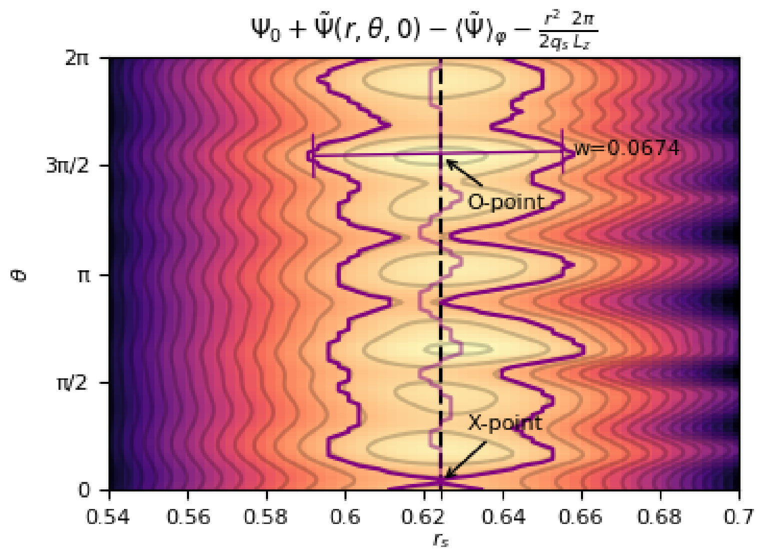


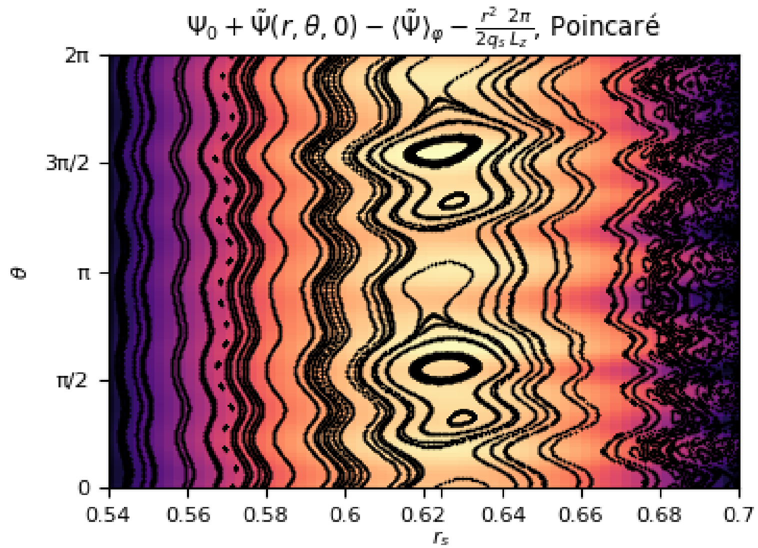
Mostly irrelevant (~static, low gradients)

However, must be taken into account when computing helical flux

$$\chi = \psi_0 + \tilde{\psi} - \frac{r^2}{2q_s}\frac{2\pi}{L_z}$$

IV. Island size and structure





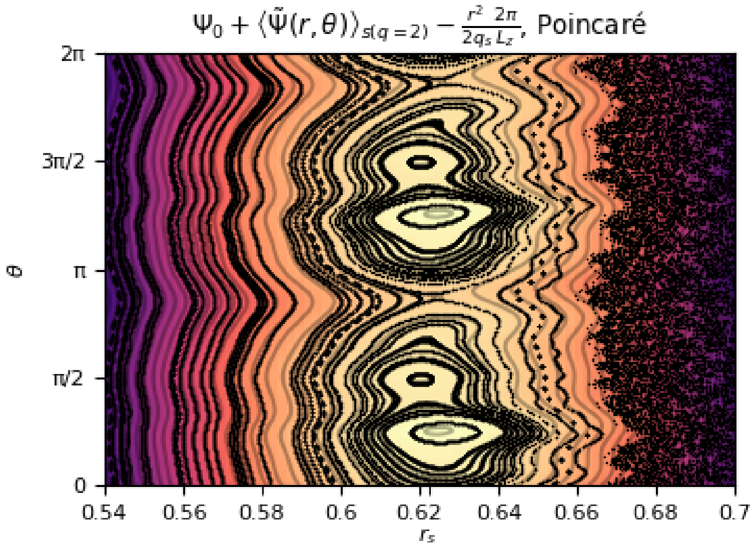
Removing axisymmetric (equilibrium) perturbation allows to recover
- overall island structure
- X-points, O-points phases
- Island width (order of magnitude)

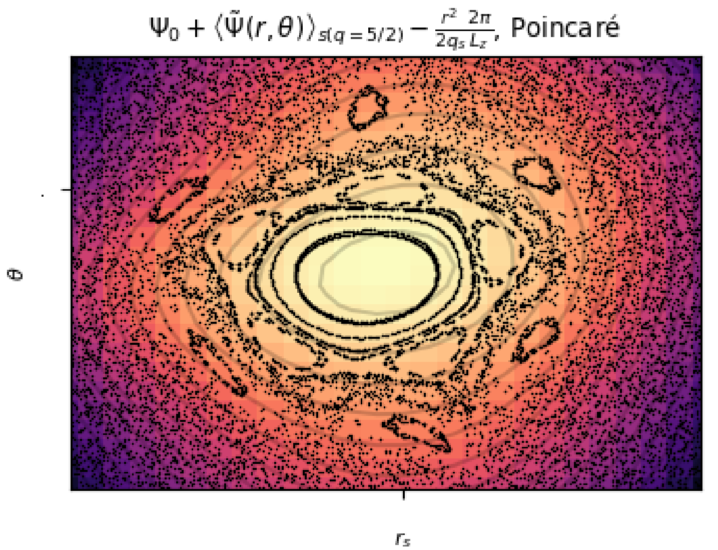
Or : only keep resonant perturbation

However that's still a 2D diagnostic
- Each φ-plane gives a different island
- KAM islands in stochastic region

Island size from Poincaré section ?

IV. Island size and structure





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Island size from Poincaré section ?

IV. Island size and structure

Island size from Poincaré map :

3 kind of field lines :

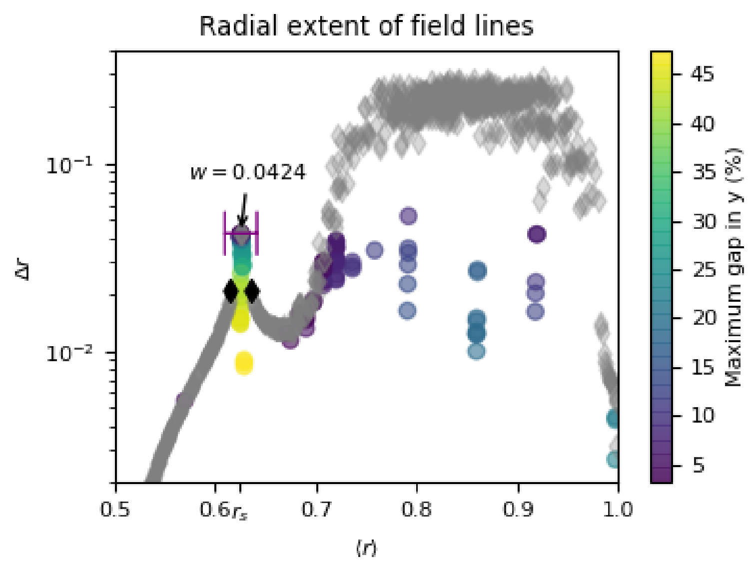
- « regular » field lines
- field lines « trapped » inside island
- stochastic region

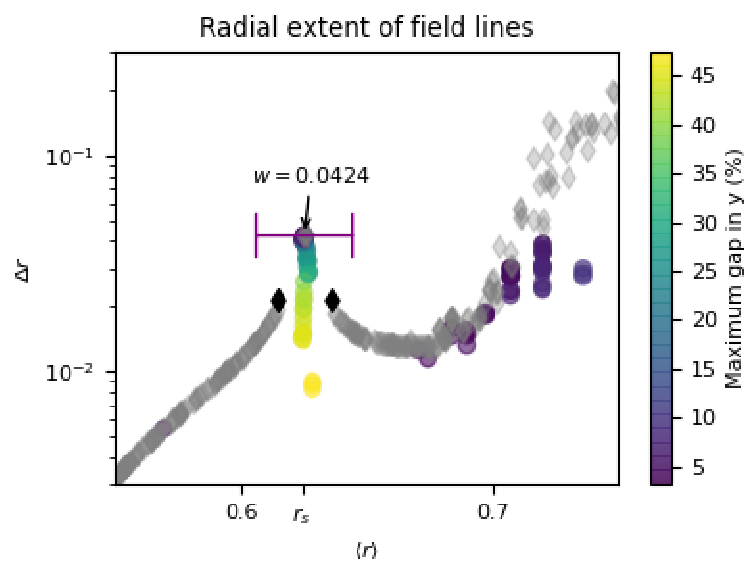
Field lines inside an island don't have θ values close to that of X point(s) → *gap in θ values*

Island size is approached by the radial width of the largest field line trapped within an island

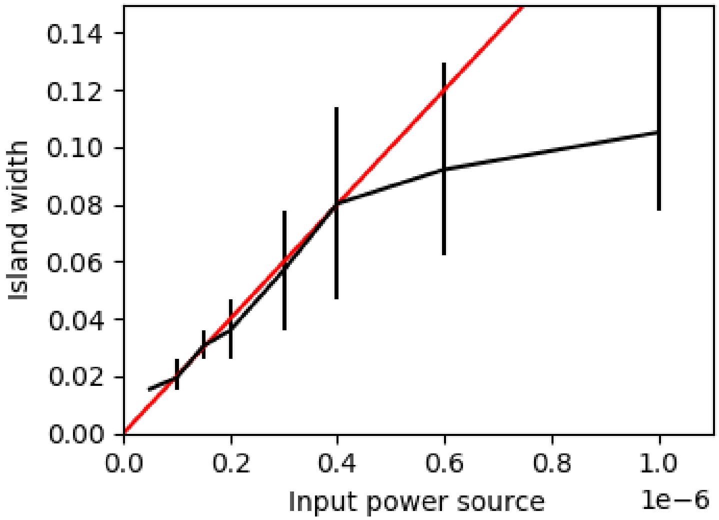
Average r value → resonant surface

Only free parameter : gap threshold





IV. Island size and structure



Island size proportional to power source

Weaker dependence after a threshold corresponding to turbulent stochastic zone spreading

→ More efficient transport for a given power source & fluctuation level ? No

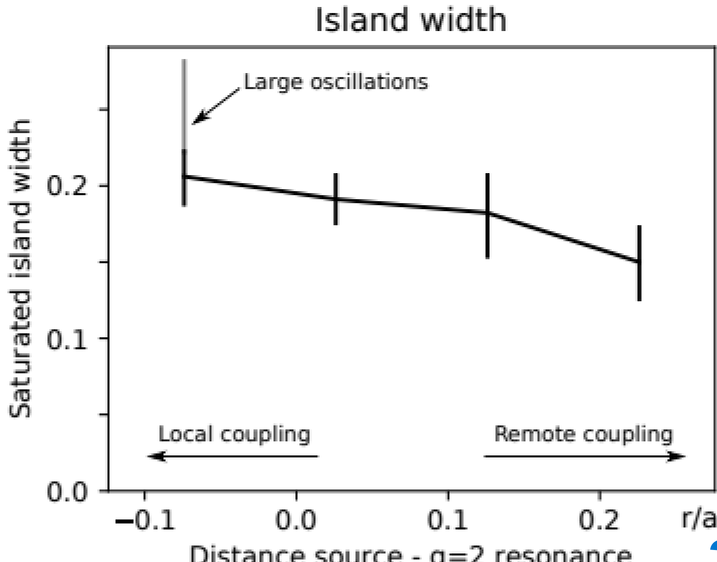
→ Stochasticity limits island size ?

Not in 2D case !

Limited dependance on distance

Same order of magnitude for local and remote coupling

Robust mechanism



IV. Island size and structure

Field lines in the stochastic region near an island can have a quasi-regular structure

- over >2500 returns to Poincaré plane
- beware numerical effects !

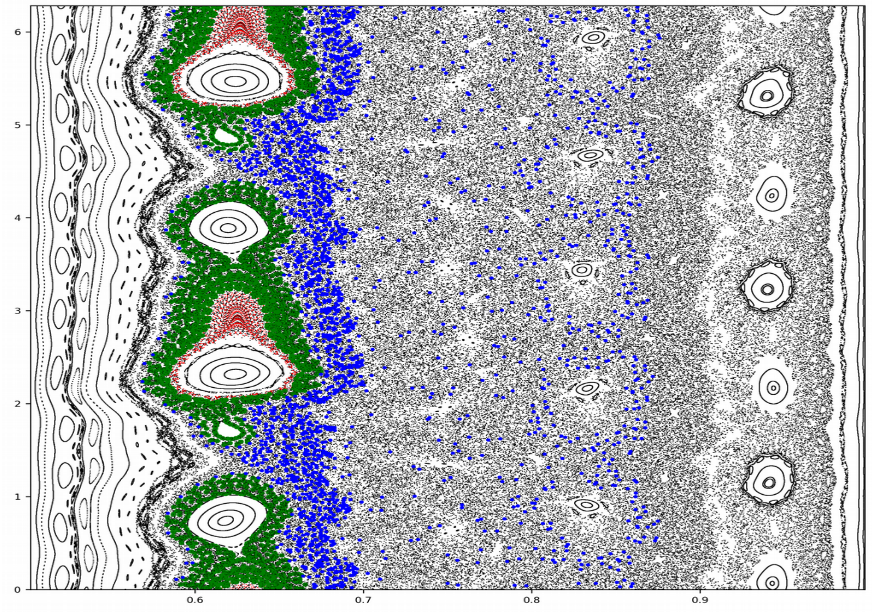
Probably not topological

field line may ultimately escape

However, relevant for transport / NTM

Correctly catalogued as island field line.

→ island size diagnostic includes green region



V. Conclusions

- A basic RMHD model has been used to investigate the interaction between **small scale interchange turbulence** and **magnetic islands** when tearing instability is marginally stable.

- In 2D slab geometry, it has been shown that small scale turbulence leads to the **NL generation of seed magnetic islands** thanks to **a interchange modes beating**. The island and the turbulence are located around the same resonant surface.

- Bootstrap current has been added in the 2D slab model. **Self-consitent generation of NTMs from turbulence driven seed island.**

- In 3D geometry, nonlinear simulations have been performed in cases where interchange instability plays at the edge whereas the q=2 surface is located in an inner area. At the edge, **efficient nonlinear beating of interchange modes generates large radial tearing modes which overlap the q=2 surface**. Thus a (2, 1) magnetic island is generated.

Numerical tool : AMON code

◆ Semi-spectral code :

Radial direction : finite difference

Poloidal and axial (for 3D) directions : spectral

<u>Resolution used for this study</u> : 1024 points in the radial direction and 256 poloidal modes

◆ Temporal scheme :

Runge-Kutta 4

◆ Boundary conditions :

Radial direction : 0 at the boundaries

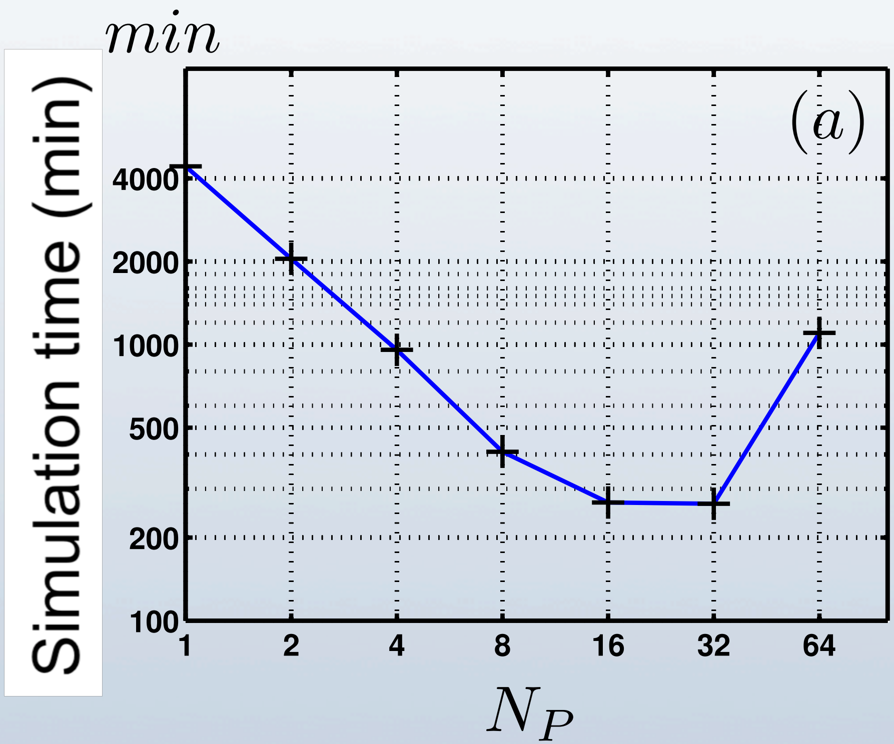
Poloidal and axial directions : periodic

◆ Nonlinear terms :

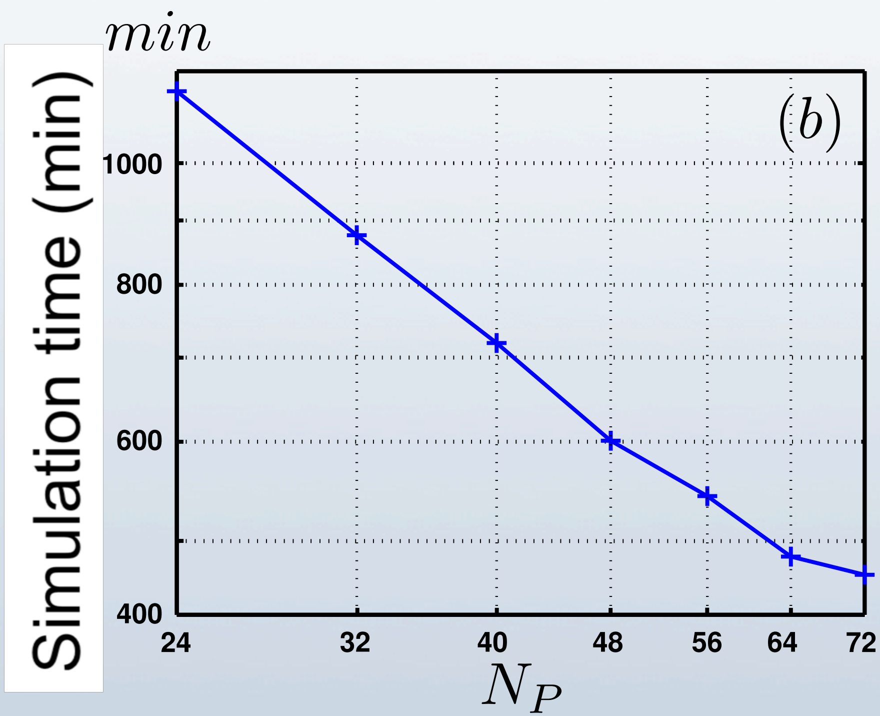
Quadratic terms conservation

Numerical tool : Performances code

- 1024*512, 2D simulations with 2 fields, 500000 iterations on Nestor (8 procs/node) :

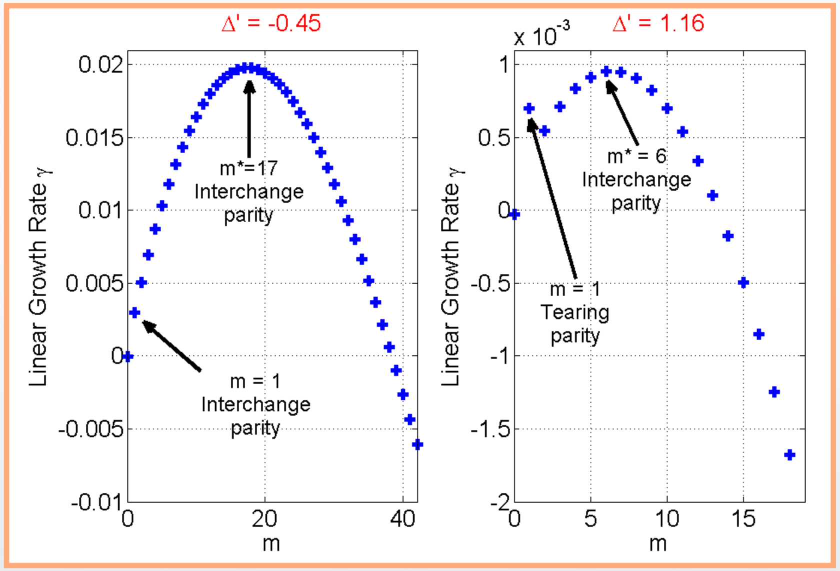


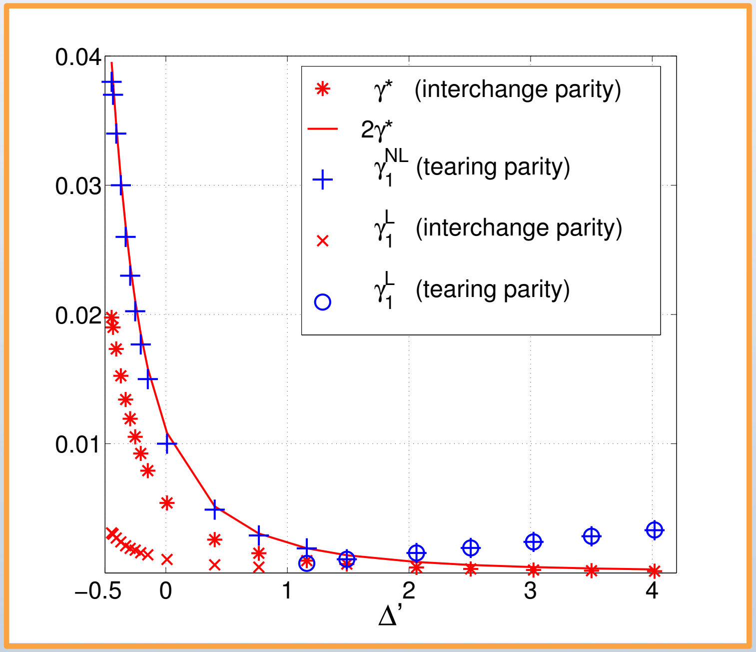
- 256*256*128, 3D simulations with 3 fields, 500000 iterations on Juelich (8 procs/node) :



Beating of interchange modes

- $\Delta' = [\frac{\mathbf{d} \ln \psi_{\text{ideal}}}{\mathbf{dx}}]_{\mathbf{0}^-}^{\mathbf{0}^+} \in [-\mathbf{0.5}, \mathbf{4.5}]$
  - Tearing marginally stable/unstable
  - Interchange instability growth rate depends on the equilibrium and therefore is linked to Δ'
  - Interchange is enhanced when Δ' becomes negative ($d\gamma_\star/d\Delta' < 0$)



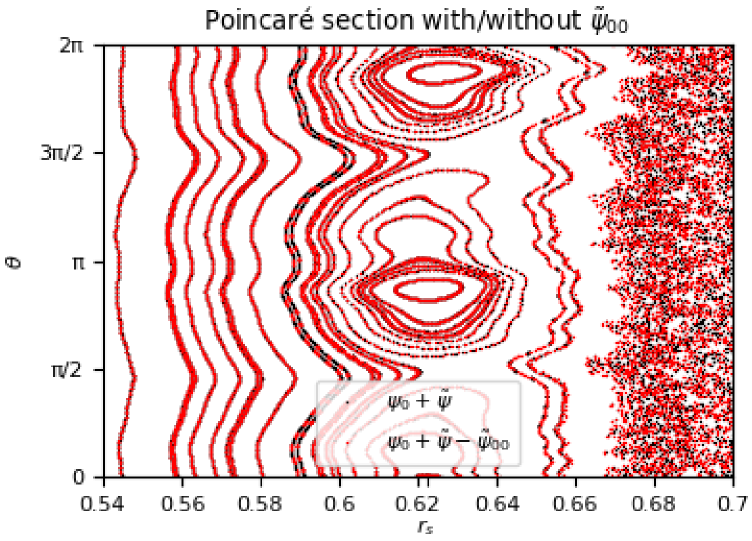


- Beating of small scales interchange modes drive large scale modes:

$2\gamma_\star > \gamma_1^L$: Interchange driven island

$2\gamma_\star < \gamma_1^L$: Tearing driven island

III. Perturbed current profile



Perturbed current profile is not enough to destabilize islands.

Island generation is not due to
tearing + anomalous resistivity

III. Profile stiffness