Anatomy of turbulent transport in energy-space

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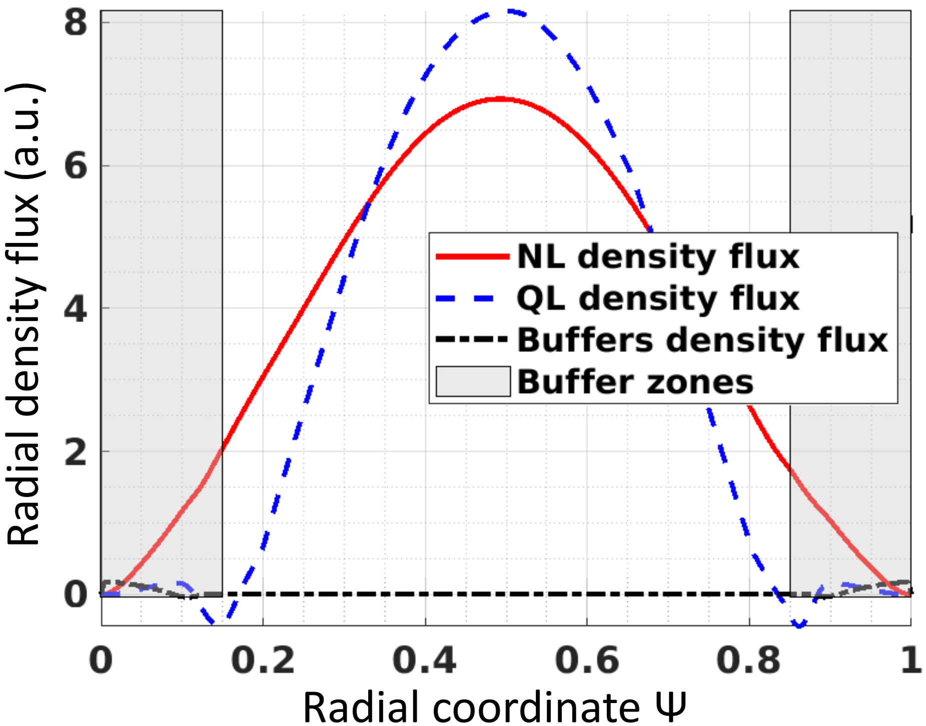
Fundings: French Research Federation for Magnetic Confined Fusion

Conclusion: quasilinear theory works!...?...?!...

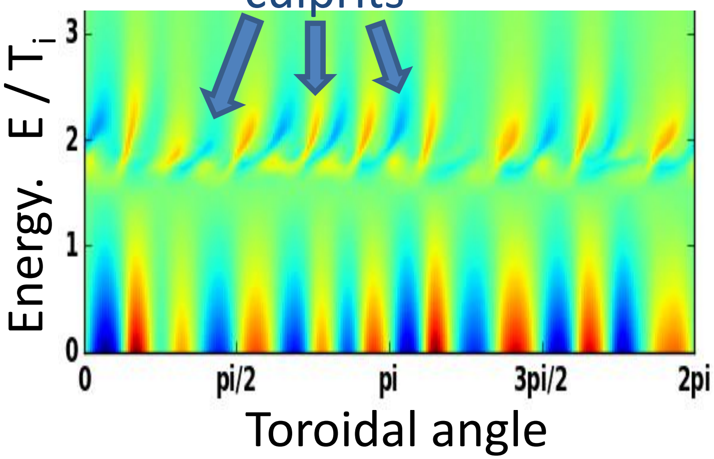
GK simulation of turbulent transport in tokamak core

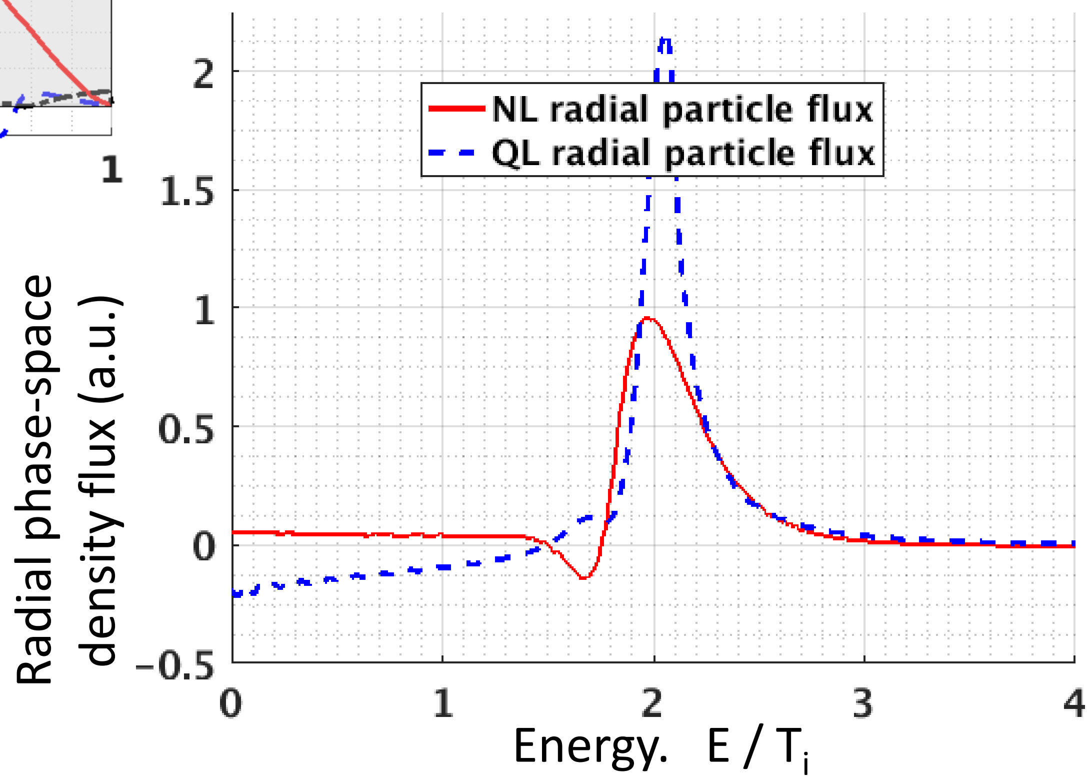


Good prediction

for the wrong reasons

culprits

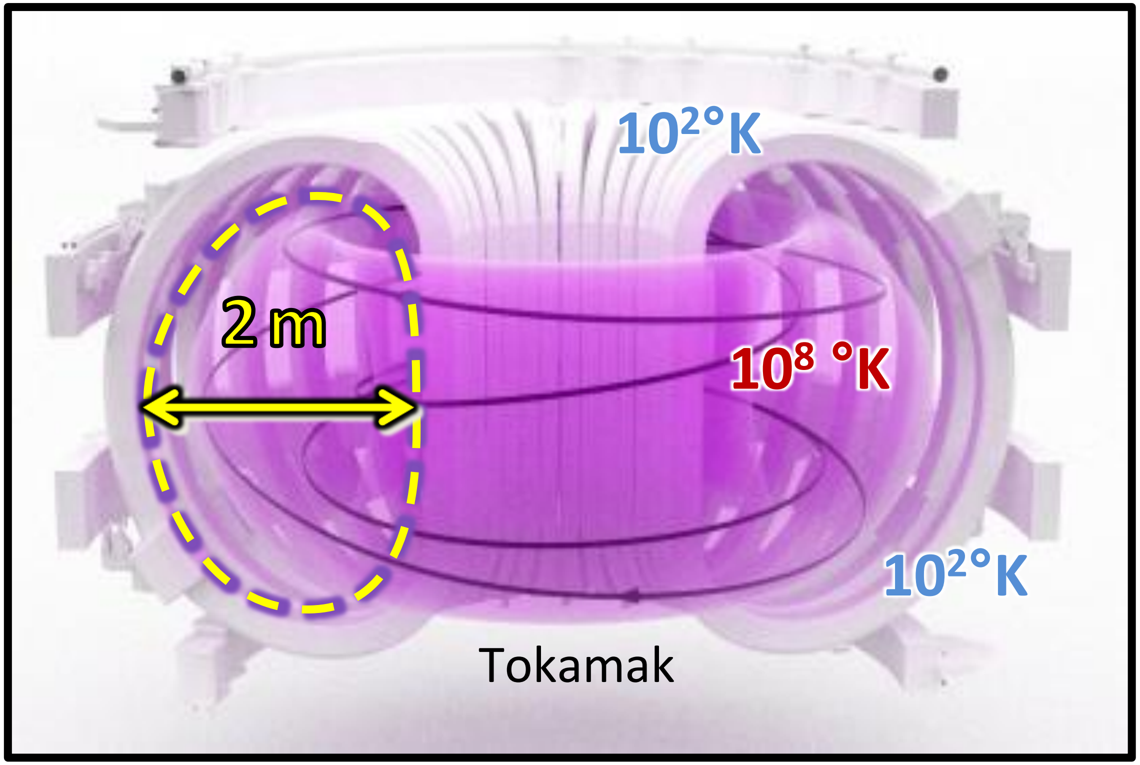




Context: Turbulence in the core of fusion plasmas

Core turbulence

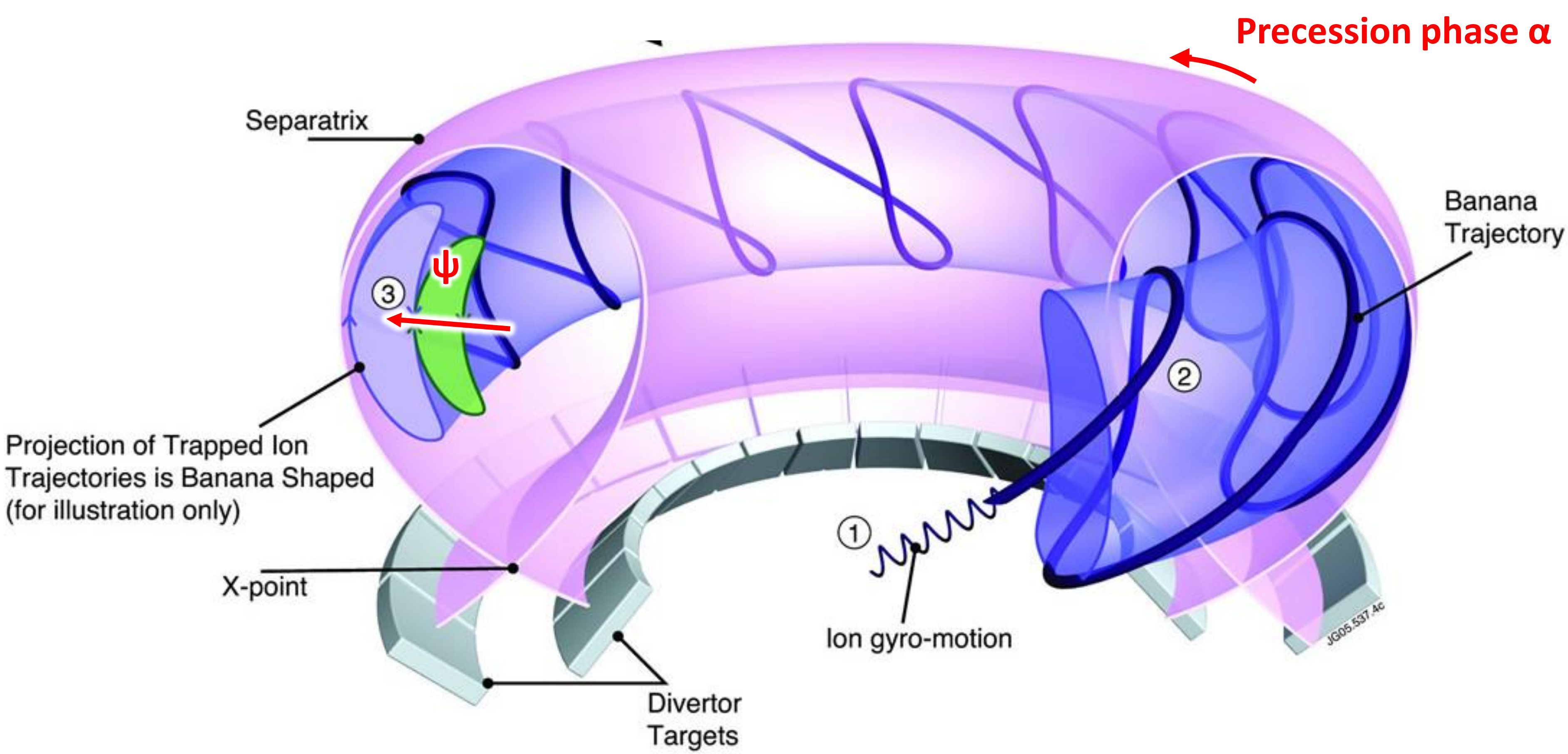
- Microscopic (mm) waves
- Driven by pressure gradients
- Dominated by Ion-Temperature-Gradient (ITG) mode and Trapped-Electron-Mode (TEM)
- Collisional mean free path (km) >> λ



- In this work: electrostatic, collisionless
- Transport particles and energy between core and edge via **E**x**B** drift

⇒ Degrade confinement
⇒ May be harnessed

Magnetically trapped particles (bananas)



Deeply trapped particles described by 3 variables: α, ψ, E

Radial coordinate

Kinetic energy

Approach: bounce-averaged gyrokinetics

Kinetics: **6D Phase-space** (3D + 3V)

$\omega \ll \omega_{gyro}$

Gyrokinetics: **4D Phase-space + 1 parameter**

$\omega \ll \omega_{bounce} \ll \omega_{gyro}$

Gyrobounce gyrokinetics: **2D Phase-space + 2 parameters**

Pitch-angle = $\pi/2$

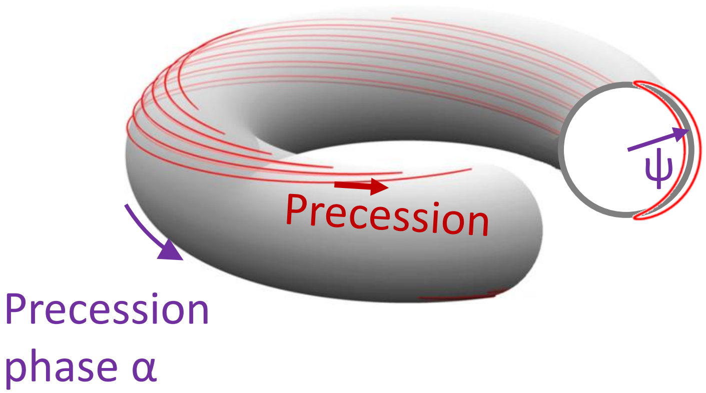
2D Phase-space + 1 parameter

(α, ψ) (Kinetic energy E)

Main limitation: $\omega \sim \omega_{precession}$

$\Rightarrow$ Trapped-Ion Mode (TIM) and Trapped-Electron Mode (TEM)

$\Rightarrow$ Between DNS and reduced model



Model equations

Vlasov equation:

$$\frac{\partial f}{\partial t} + \underbrace{[J_0\phi, f]_{\alpha,\psi}}_{\text{E x B motion}} + \underbrace{E\Omega_d\frac{\partial f}{\partial \alpha}}_{\text{Precession}} = 0$$

$$\dot{\alpha} = E\Omega_d + \frac{\partial \bar{\bar{\phi}}}{\partial \psi}$$

$$\dot{\psi} = -\frac{\partial \bar{\bar{\phi}}}{\partial \alpha}$$

Quasi-neutrality:

$$\underbrace{C_1\left[\phi - \langle\phi\rangle + \mathcal{F}^{-1}(\imath\delta_m\hat{\phi}_m)\right]}_{\delta n_{passing}} - \underbrace{C_2\bar{\Delta}\phi}_{\text{Polarisation}} = \underbrace{\frac{2}{\sqrt{\pi}}\int_0^\infty J_0(E) f\sqrt{E}\,\mathrm{d}E \; - \; 1}_{\bar{\bar{\delta n}}_{trapped}}$$

$C_2\bar{\Delta}\phi$: $\sim (\delta_b k_r)^2$

$J_0(E)$: GK average ≈ 1

Depret '00

Tagger, Laval, Pellat '77 *Sarazin '05*

Biglari, Diamond, Terry '88 *Darmet '08*

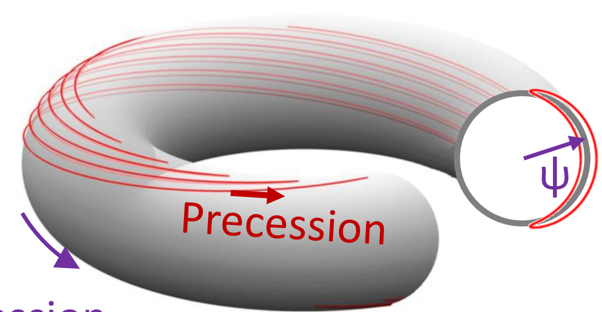
$\Rightarrow$ TERESA code

Cartier-Michaud '13

Drouot '14

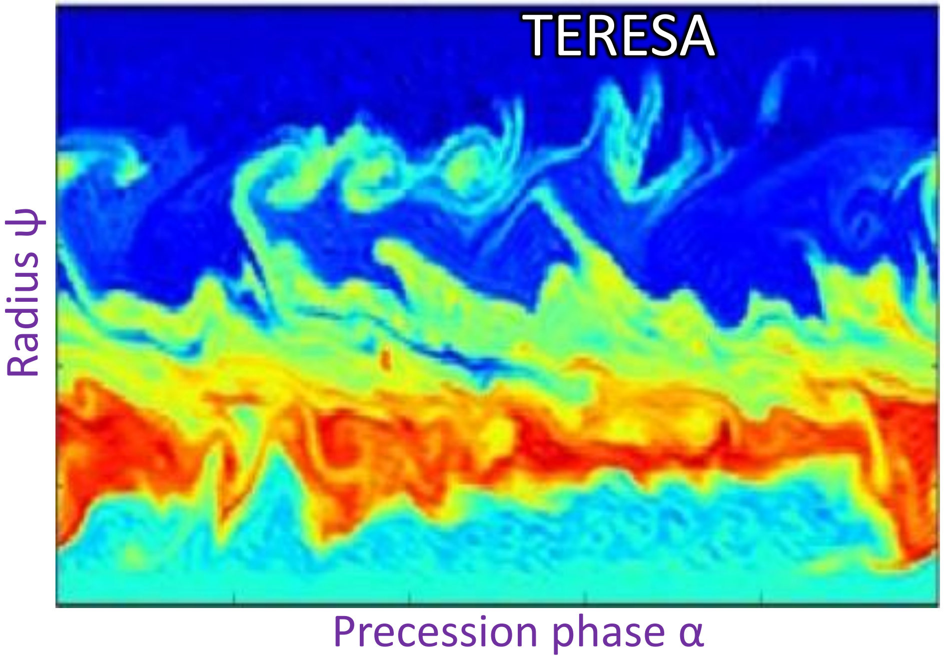
- Global, full-f, fixed-gradient (flux-driven soon)
- Collisionless, electrostatic
- Quasi-adiabatic passing particles

Lesur '17

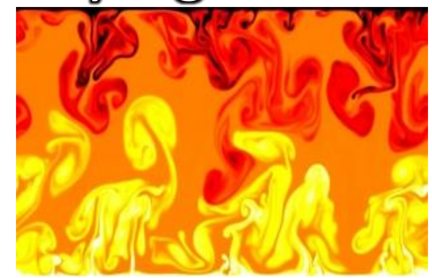


TERESA explores kinetic effects in tokamak turbulence

Kinetic version of classic instabilities

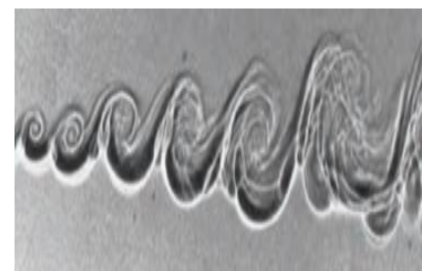


Rayleigh-Bénard

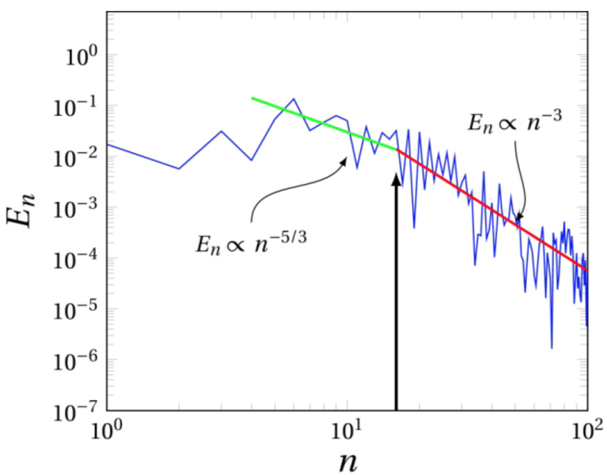


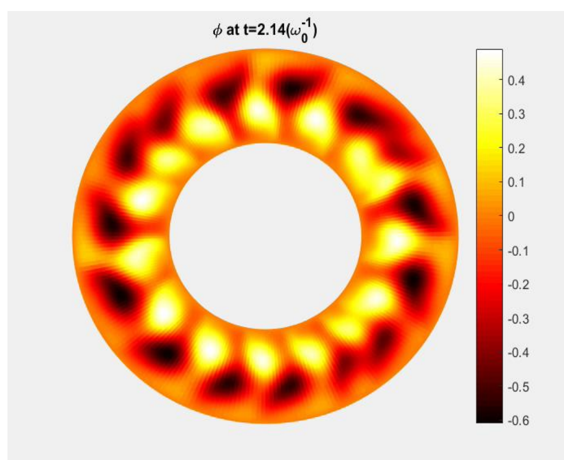
Cartier-Michaud '14

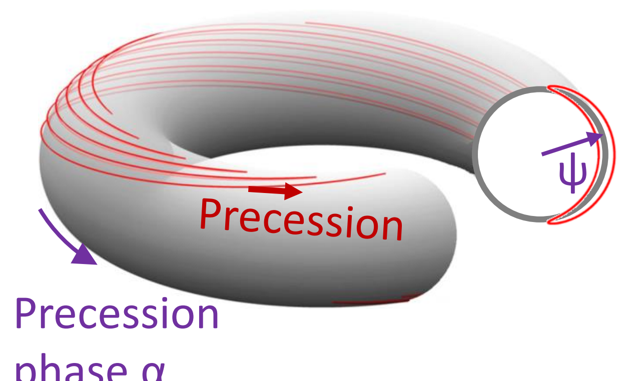
Kelvin-Helmholtz



Palermo '15







Derivation of quasilinear theory

Vlasov in Fourier space

$$\frac{\partial \hat{f}_m}{\partial t} + \underbrace{im\frac{E\Omega_d}{Z}\hat{f}_m}_{\text{Free streaming (precession)}} - \underbrace{im\hat{\phi}_m\frac{\partial \hat{f}_0}{\partial \psi}}_{\text{Linear drive}} = \underbrace{-\sum_l il\hat{f}_l\frac{\partial \hat{\phi}_{m-l}}{\partial \psi} + \sum_{l\neq 0} i(m-l)\hat{\phi}_{m-l}\frac{\partial \hat{f}_l}{\partial \psi}}_{\text{Nonlinear terms}}.$$

Radial flux (phase dynamics)

$$m=0 \qquad \frac{\partial \langle f \rangle}{\partial t} + \sum_l il\frac{\partial (\hat{f}_l\hat{\phi}_l^*)}{\partial \psi} = 0$$

$\Rightarrow$ Evolution of mean fields

$$m\neq 0 \qquad \frac{\partial \hat{f}_m}{\partial t} + im\frac{E\Omega_d}{Z}\hat{f}_m - im\hat{\phi}_m\frac{\partial \langle f \rangle}{\partial \psi} + im\hat{f}_m\frac{\partial \hat{\phi}_0}{\partial \psi} = 0$$

Neglecting all nonlinearities except for $l=m$

Derivation of quasilinear theory (2)

$$m \neq 0 \qquad \frac{\partial \hat{f}_m}{\partial t} + im\frac{E\Omega_d}{Z}\hat{f}_m - im\hat{\phi}_m \frac{\partial \langle f \rangle}{\partial \psi} + im\hat{f}_m \frac{\partial \hat{\phi}_0}{\partial \psi} = 0$$

$L\hat{f}_m$

$\Rightarrow$ Linear response

$$\hat{f}_m(\psi, E, t) = \int_0^t e^{i\omega_{R,m}(s-t)}\, im\, \hat{\phi}_m(\psi, E, s)\, \frac{\partial \langle f \rangle}{\partial \psi}(\psi, E, s)\, \mathrm{d}s.$$

$$\omega_{R,m}(\psi, E, t) = m\left(\frac{\Omega_d E}{Z} + \frac{\partial \hat{\phi}_0}{\partial \psi}\right)$$

Doppler-shift by zonal flow

Vedenov, Velikov, Sagdeev '61
Drummond & Pines '62
Sagdeev & Galeev '69

Derivation of quasilinear theory (3)

Linear theory

$$\hat{\phi}_m(\psi, E, t) = \hat{\phi}_m(\psi, E, 0)\exp\left[\int_0^t \left(-i\omega_m + \gamma_m(t')\right)\mathrm{d}t'\right]$$

$$\frac{\partial\langle f\rangle}{\partial t} = \frac{\partial}{\partial\psi}\left[\sum_l l^2|\hat{\phi}_l(\psi, E, t)|^2 \int_0^t e^{i(\omega_{R,l}-\omega_l)(s-t)}\frac{\partial\langle f\rangle}{\partial\psi}\bigg|_s \times \exp\left(\int_t^s \gamma(t')\mathrm{d}t'\right)\mathrm{d}s\right]$$

$\frac{\partial\langle f\rangle}{\partial\psi}\big|_s \to \partial_\psi\langle f\rangle|_t$

Phase-mixing for t-s > growth, relaxation times

$\int_t^s \gamma(t')\mathrm{d}t' \to \gamma(t)(s-t)$

$$\Rightarrow \quad \frac{\partial\langle f\rangle}{\partial t} = \frac{\partial}{\partial\psi}\left[D_{\mathrm{QL}}\frac{\partial\langle f\rangle}{\partial\psi}\right]$$

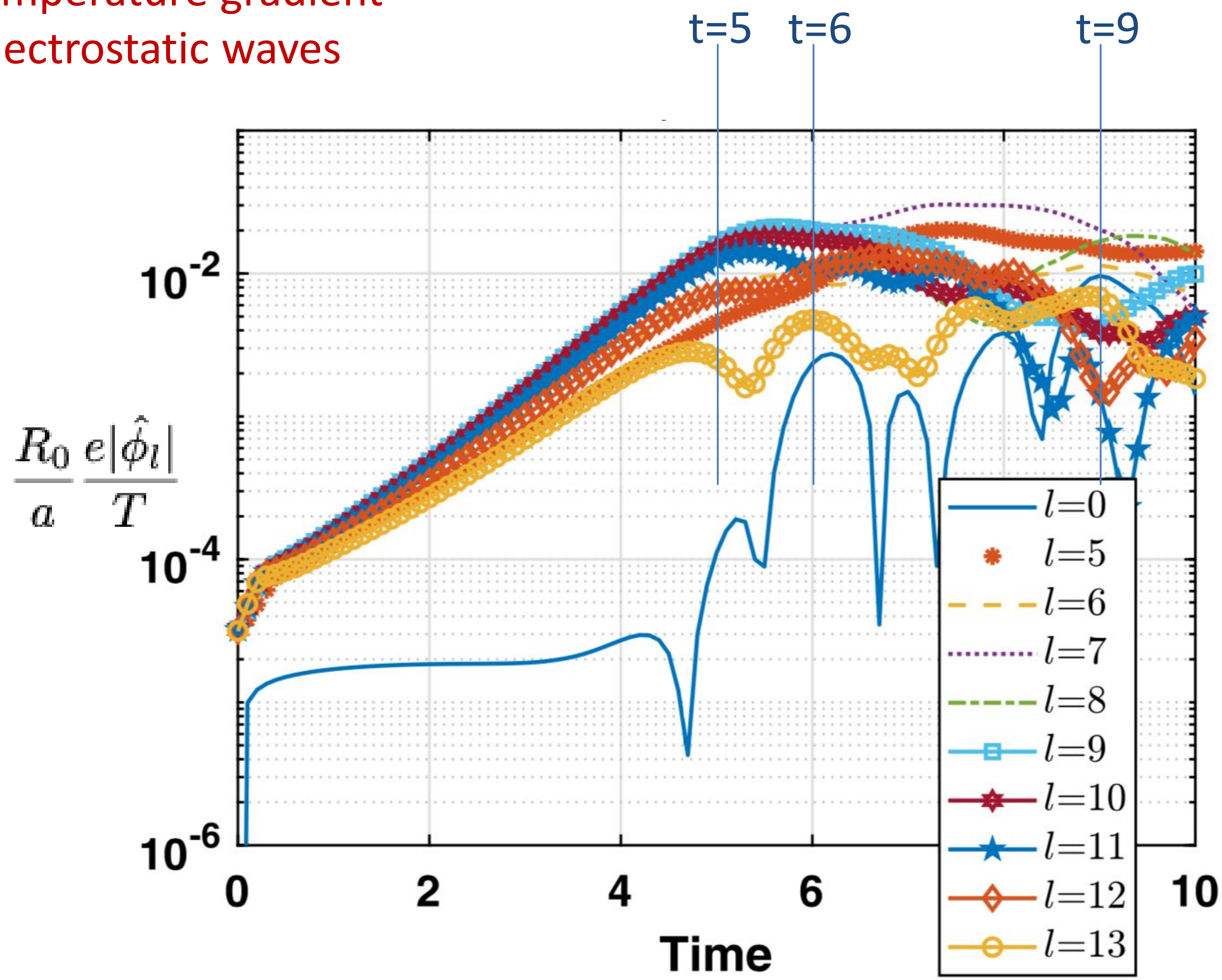
$D_{\mathrm{QL}}\frac{\partial\langle f\rangle}{\partial\psi} = \Lambda_\psi^{\mathrm{QL}}(\psi, E, t)$

Vedenov, Velikov, Sagdeev '61
Drummond & Pines '62
Sagdeev & Galeev '69

$$D_{\mathrm{QL}}(\psi, E, t) = \sum_l l^2|\hat{\phi}_l(\psi, E, t)|^2 \frac{1 - e^{-i(\omega_{R,l}-\omega_l)t-\gamma_l t}}{i(\omega_{R,l}-\omega_l)+\gamma_l}$$

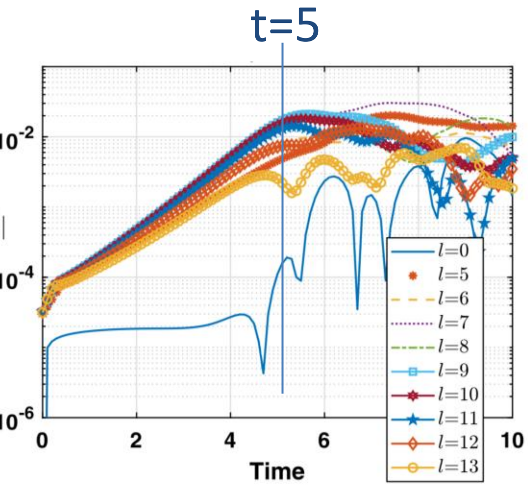
Trapped-ion-driven turbulence

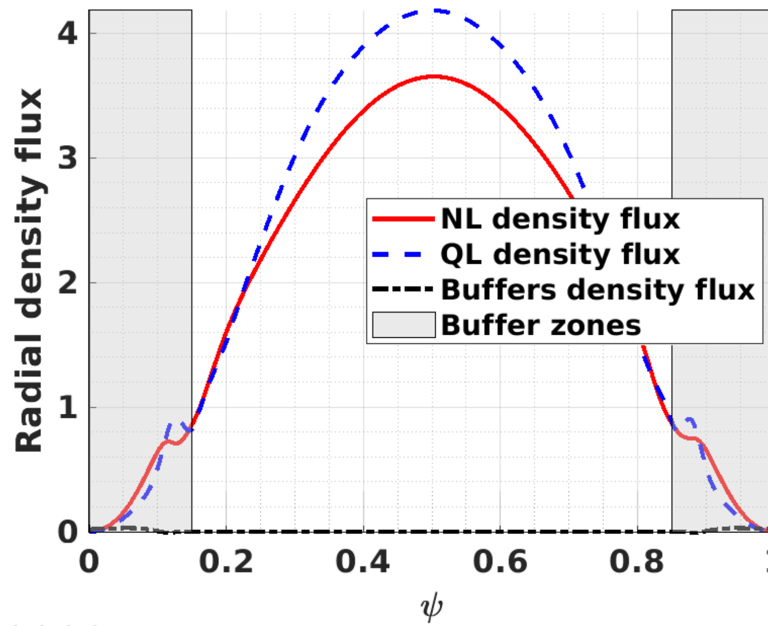
Initial temperature gradient drives electrostatic waves



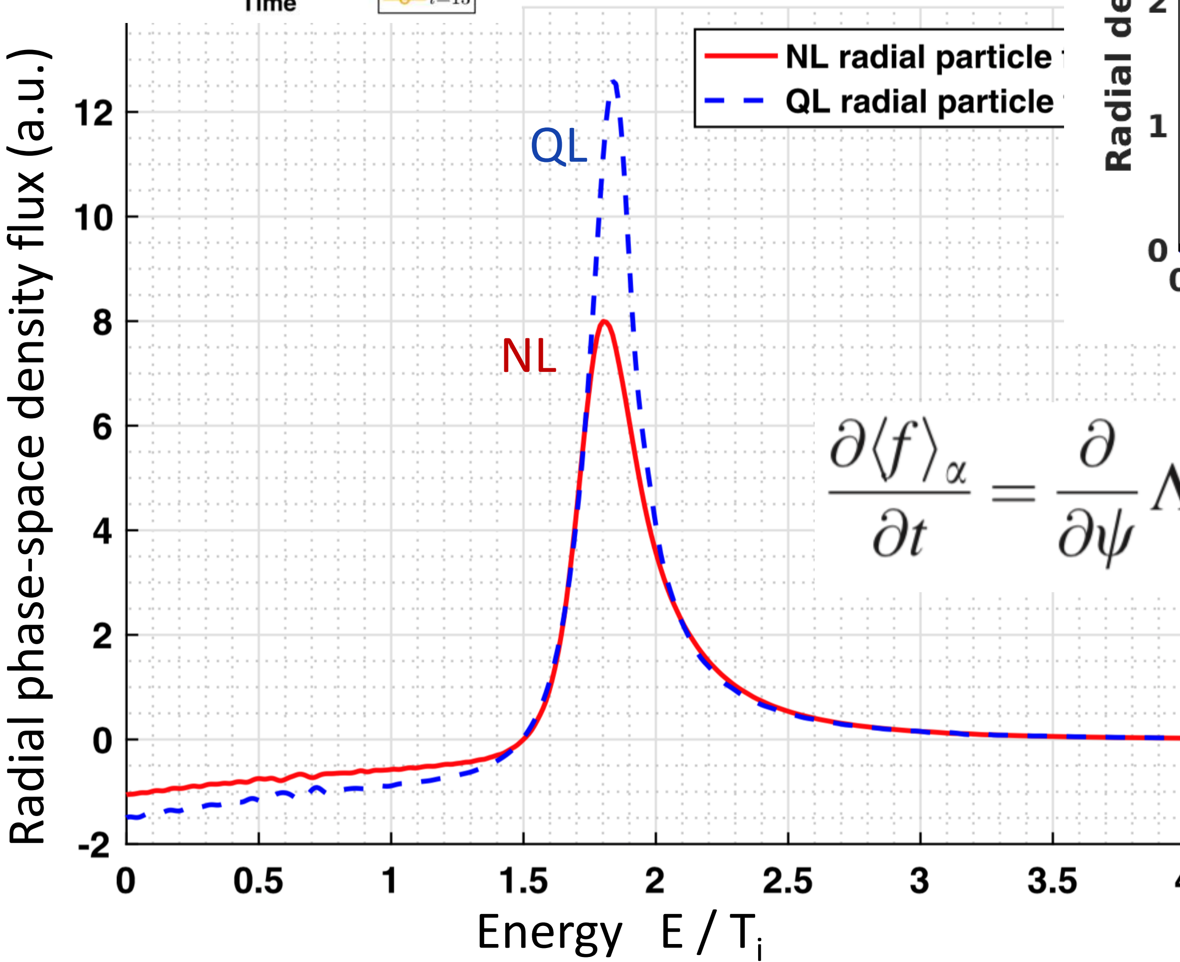
(in units of thermal precession period)

Anatomy of transport





Medina'18



$$\frac{\partial \langle f \rangle_\alpha}{\partial t} = \frac{\partial}{\partial \psi} \Lambda_\psi^{\mathrm{NL}}(\psi, E, t)$$

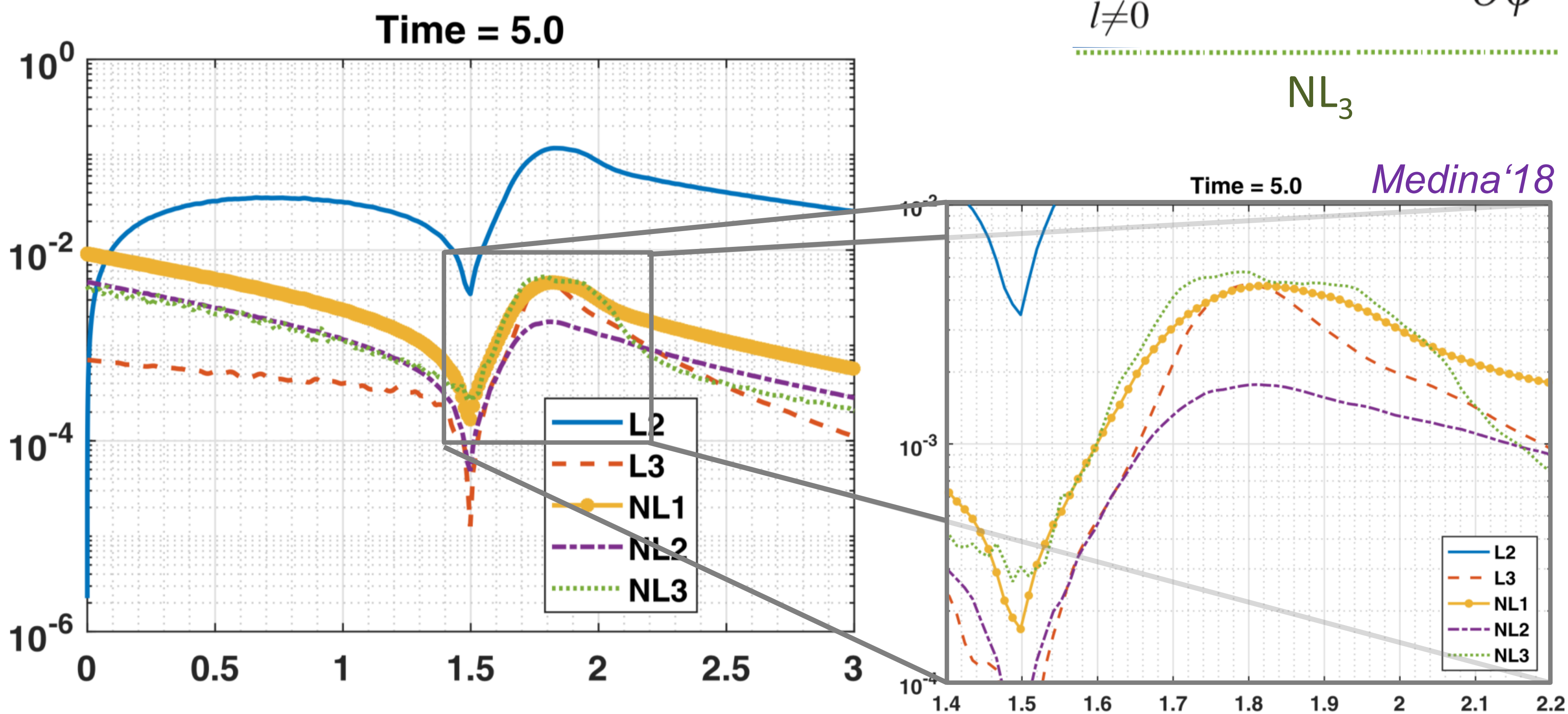
Radial phase-space density flux

Neglected NL terms are important in magnitude

$$\underbrace{\frac{\partial \hat{f}_m}{\partial t}}_{L_1} + \underbrace{im\frac{E\Omega_d}{Z}\hat{f}_m}_{L_2} - \underbrace{im\hat{\phi}_m\frac{\partial \hat{f}_0}{\partial \psi}}_{L_3} = -\sum_l il\hat{f}_l\frac{\partial \hat{\phi}_{m-l}}{\partial \psi} + \underbrace{\sum_{l\neq 0} i(m-l)\hat{\phi}_{m-l}\frac{\partial \hat{f}_l}{\partial \psi}}_{NL_3}.$$

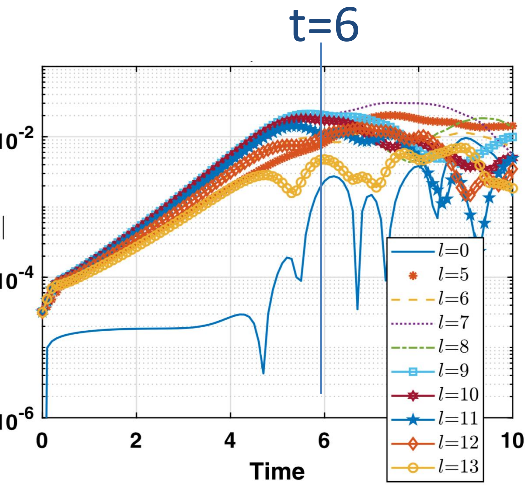
NL_1 (l=m)

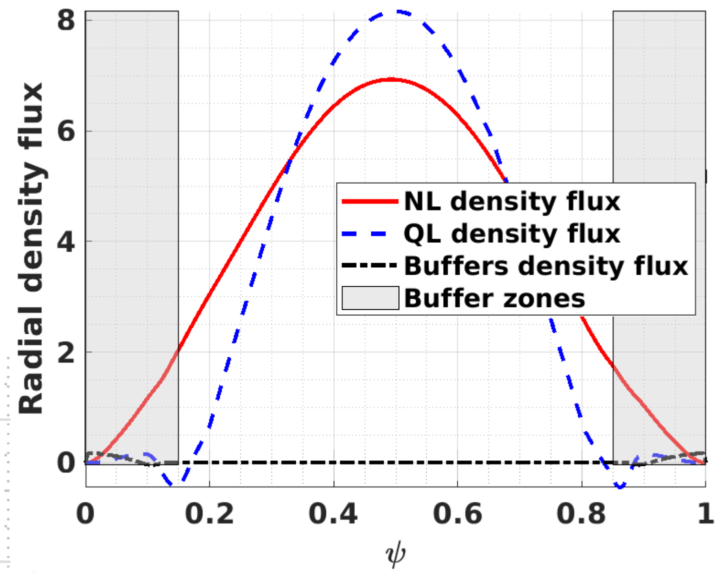
NL_2 (l≠m)

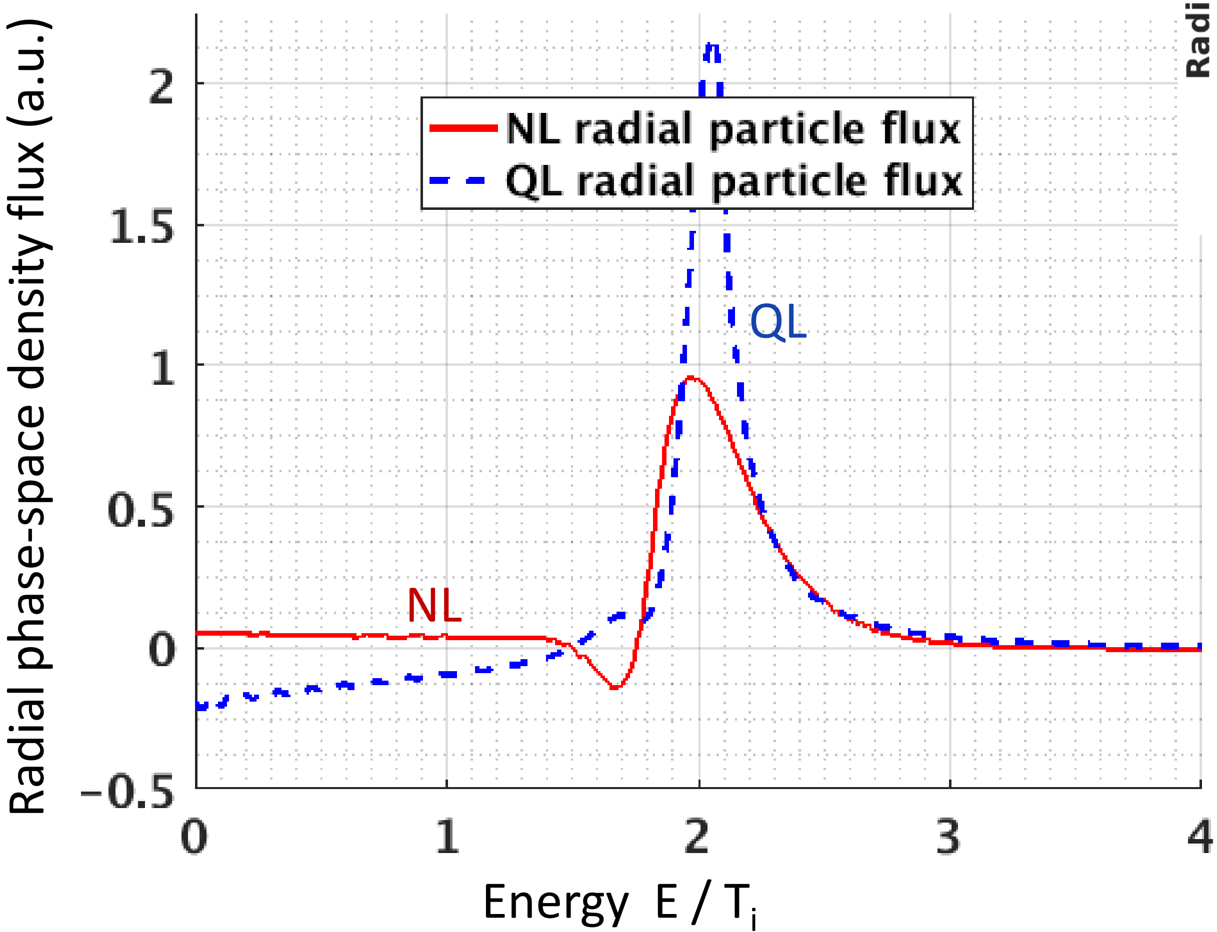


Medina'18

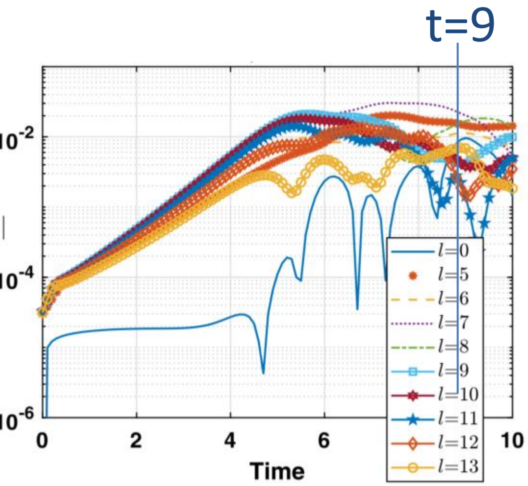
Anatomy of transport (2)

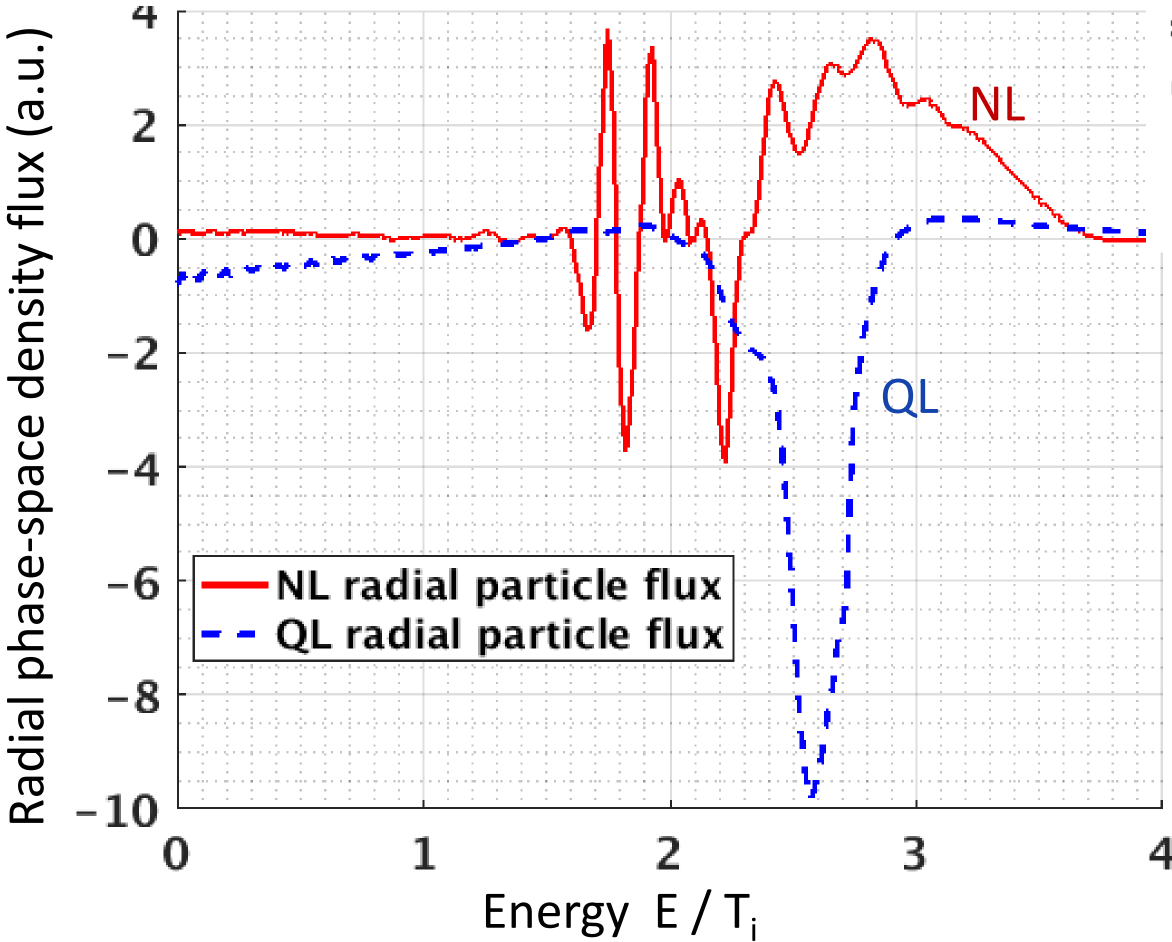


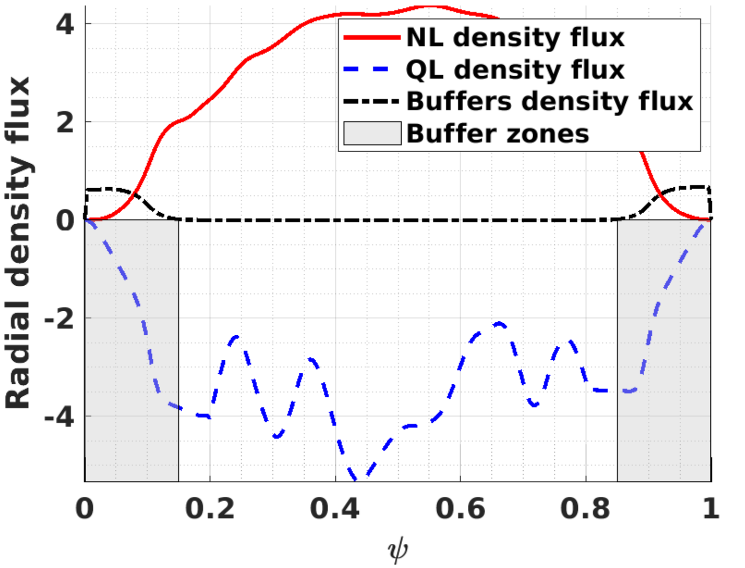




Anatomy of transport (3)

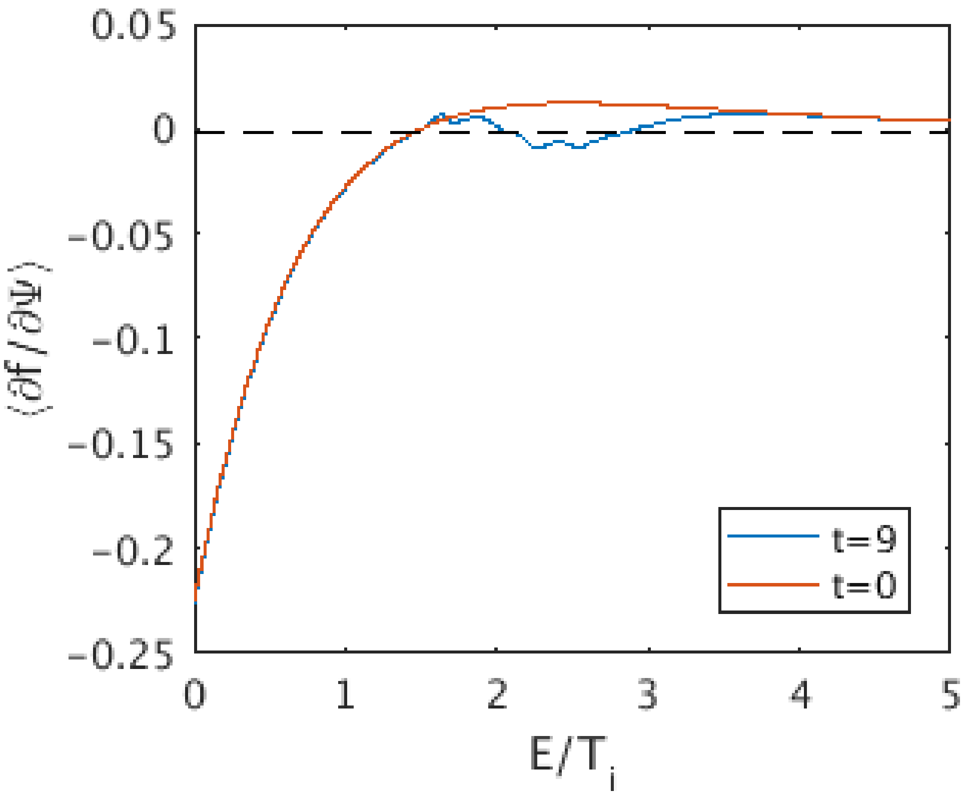


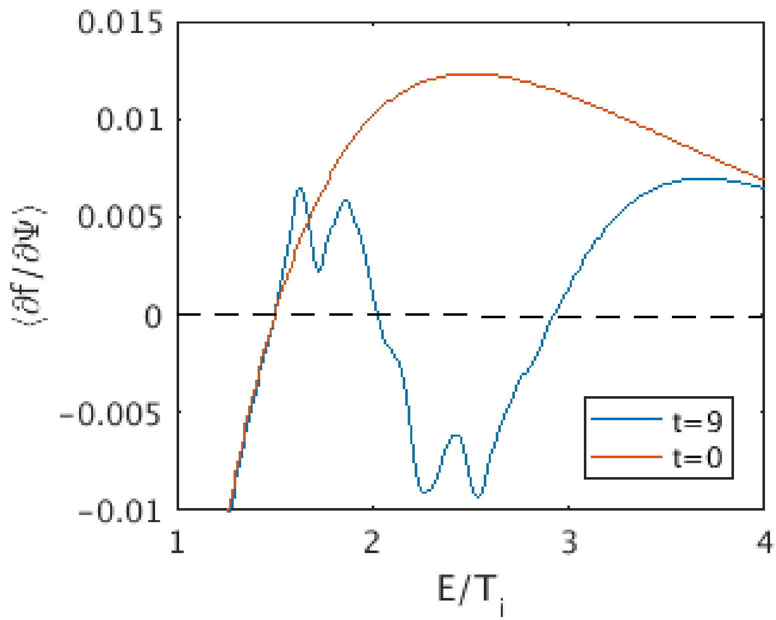




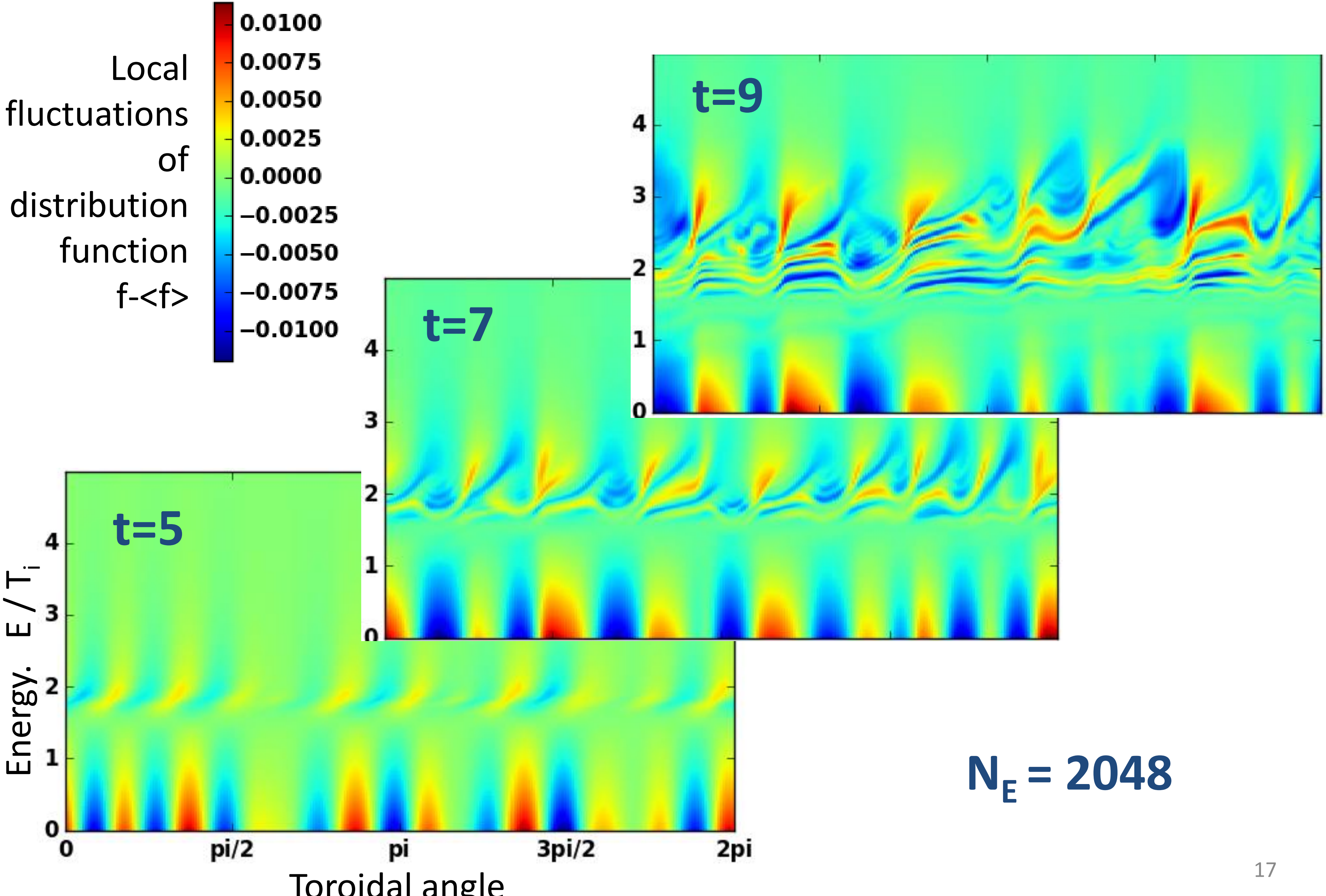
QL flux < 0 due to fine scales in energy-space

$$\Lambda_{\psi}^{\mathrm{QL}}(\psi, E, t) = -D_{QL}(\psi, E, t)\,\frac{\partial \langle f \rangle_{\alpha}}{\partial \psi}(\psi, E, t)$$





Structures in phase-space



$N_E = 2048$

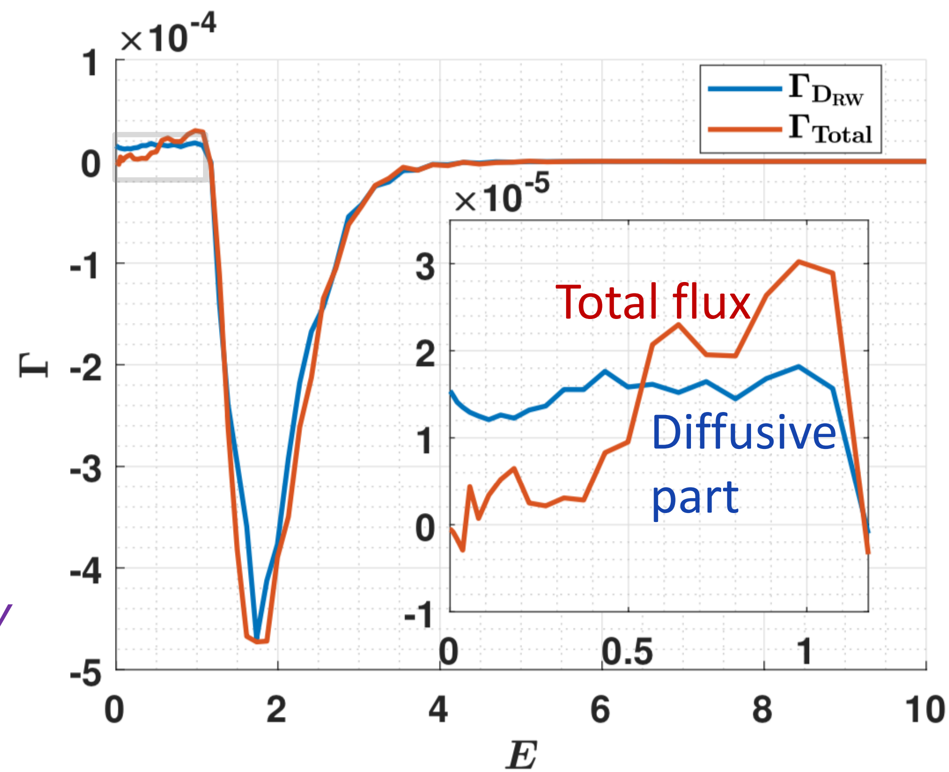
Summary

Collisionless turbulent transport is dominated by resonant processes

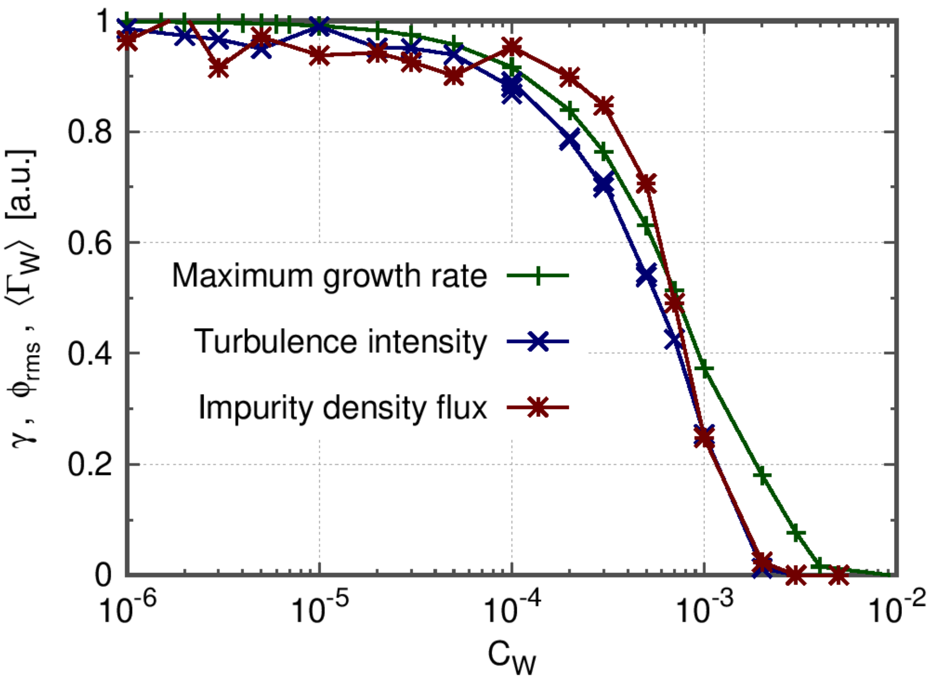
- Radial flux of particles dominated by a narrow peak around resonance
- Caveat: trapped-particle-driven only (strong resonances)
- Early nonlinear stage: diffusion in agreement with quasilinear theory
- Later stage: departure from quasilinear formalism due to small structures in energy-space

Perspectives

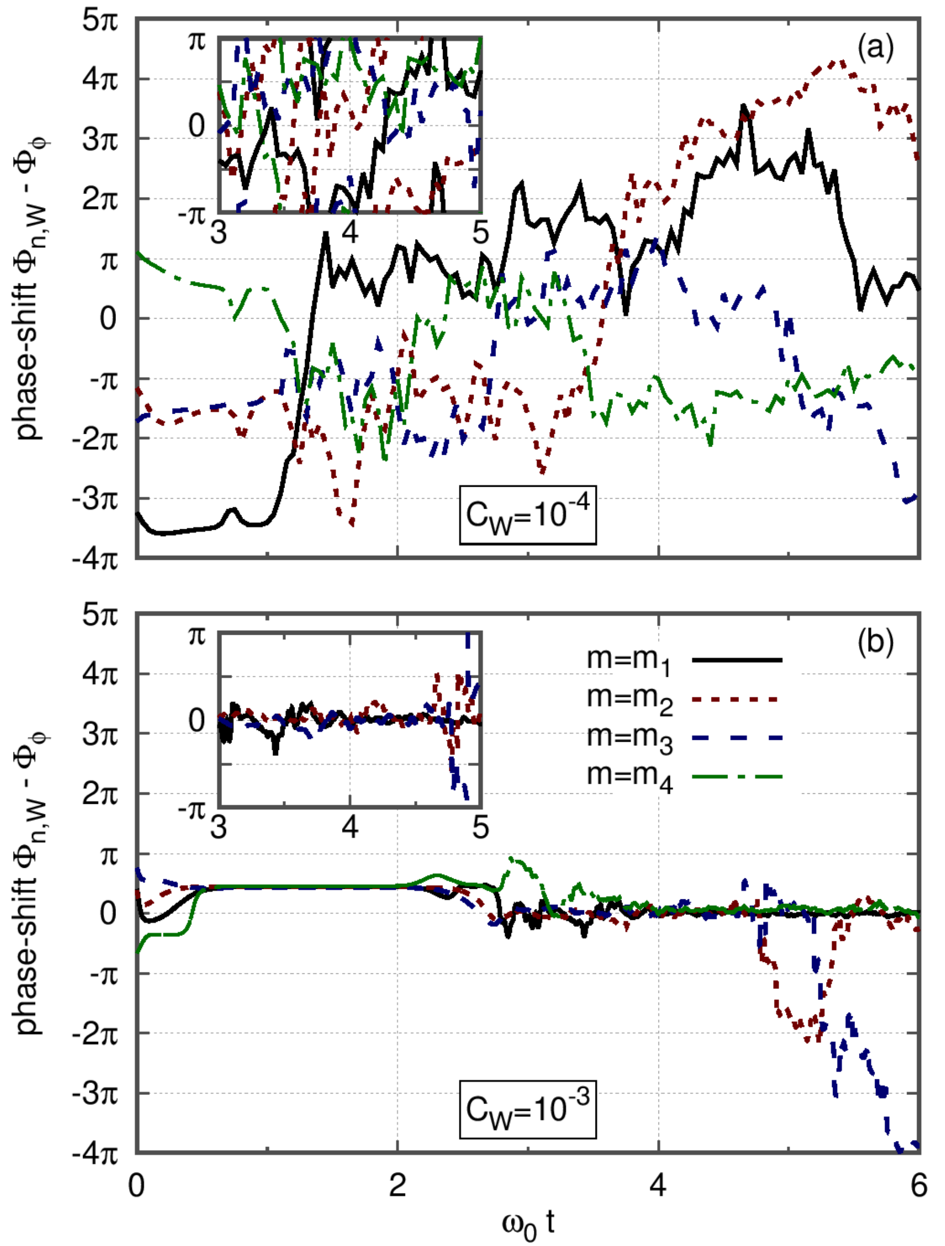
- Weak vs stronger turbulence?
- Diffusion vs convection vs trapping?
- Steady-state in a stirred system (flux-driven)?
- Collisions → *Artur Kryzhanovskyy*
- Beyond QL theory?



Bonus on phase dynamics: impurity transport



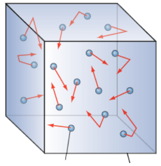
Tungsten concentration



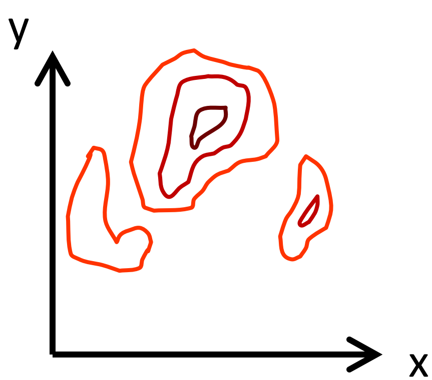
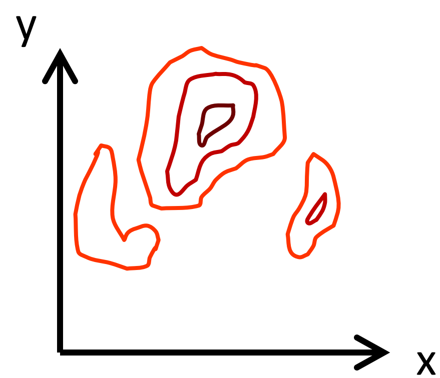
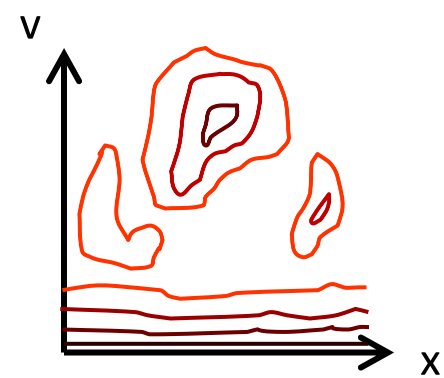
⇒ Quenching of impurity transport by synchronisation of potential fluctuations on impurity density fluctuations

Threshold?

Additional material

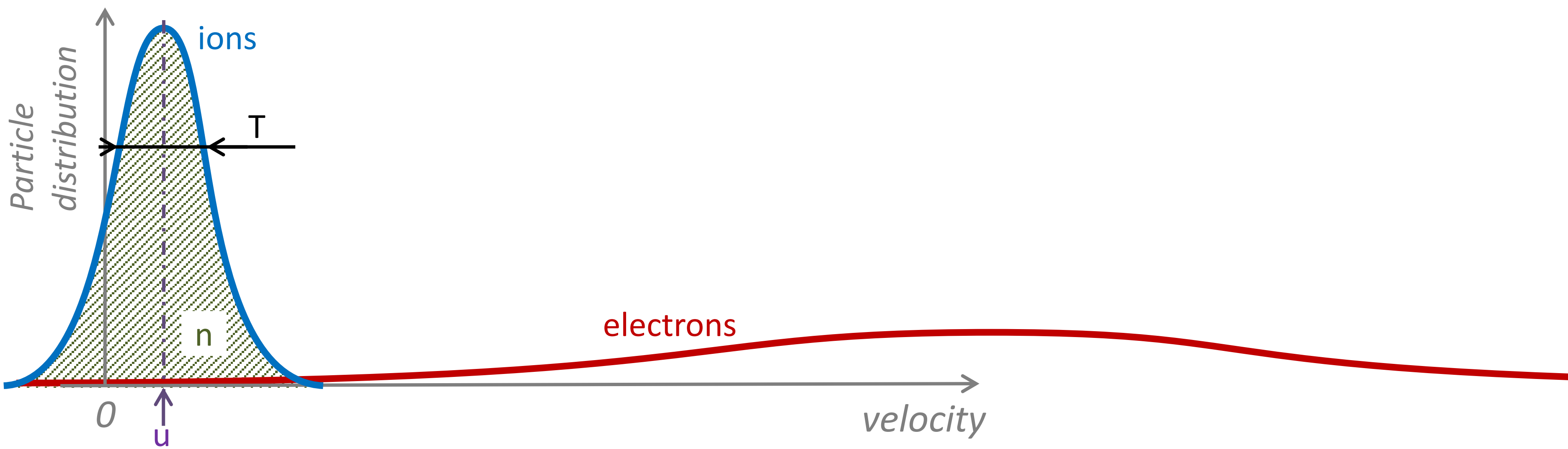


Kinetic theory

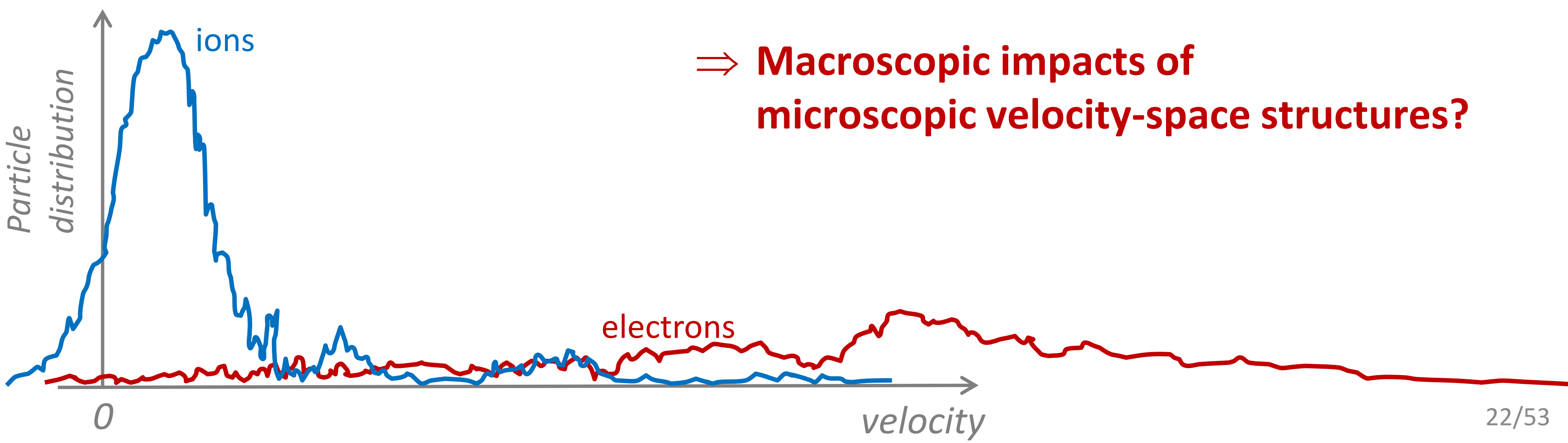
System	Many incompress. fluids in 2D	Quasi-geostrophic fluid	1D collisionless plasma
Distribution	$n(x,y,t)$	$\omega(x,y,t)$	$f(x,v,t)$
Description space	2D config. space 	2D config. space 	2D phase space 
Continuity equation	$\frac{\partial n}{\partial t} + u_x \frac{\partial n}{\partial x} + u_y \frac{\partial n}{\partial y} = 0$	$\frac{\partial \omega}{\partial t} + u_x \frac{\partial \omega}{\partial x} + u_y \frac{\partial \omega}{\partial y} = 0$	$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + E \frac{\partial f}{\partial v} = 0$
Self-consistency	Hierarchy of fluid equations + closure	Stream function $\omega = \nabla^2 \psi$	Poisson $\frac{\partial E}{\partial x} = \sum_s q_s \int f_s \, dx$

Fluid vs kinetic

1D fluid model: density n(x, t), mean velocity u(x, t), and temperature T(x, t)



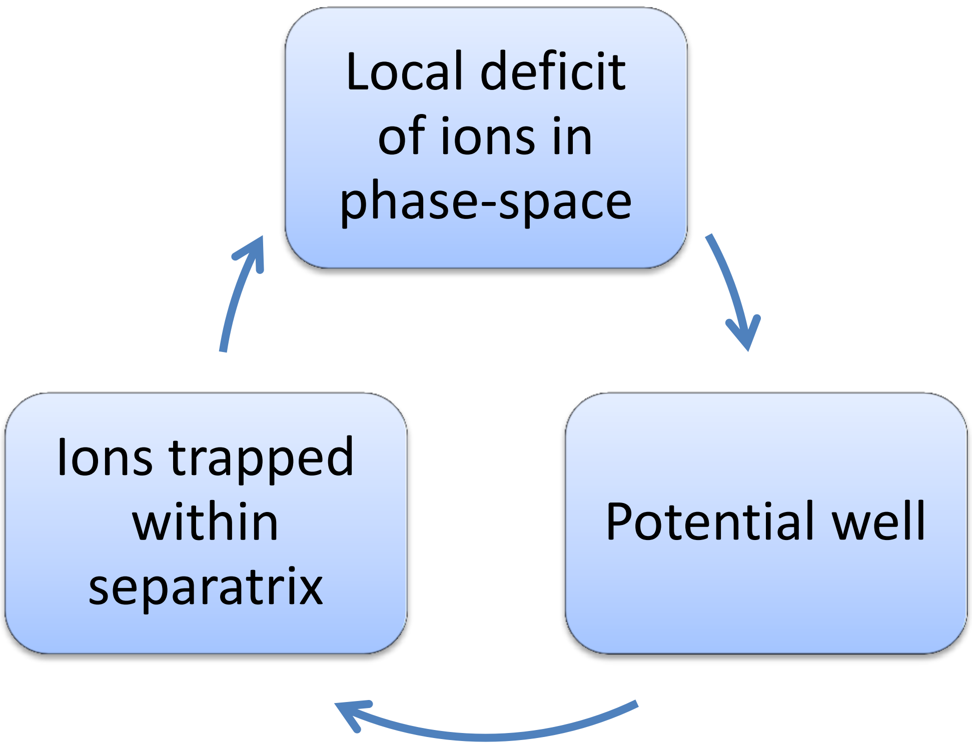
1D kinetic model: fonction de distribution f(x, v, t) => 2D phase-space

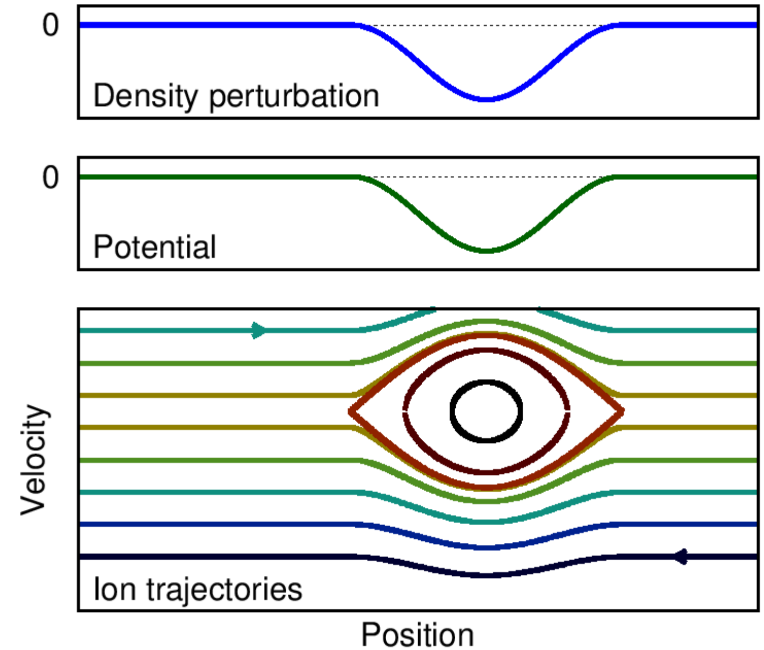


$\Rightarrow$ **Macroscopic impacts of microscopic velocity-space structures?**

Electrostatic trapping yields phase-space vortex

Self-sustaining structure

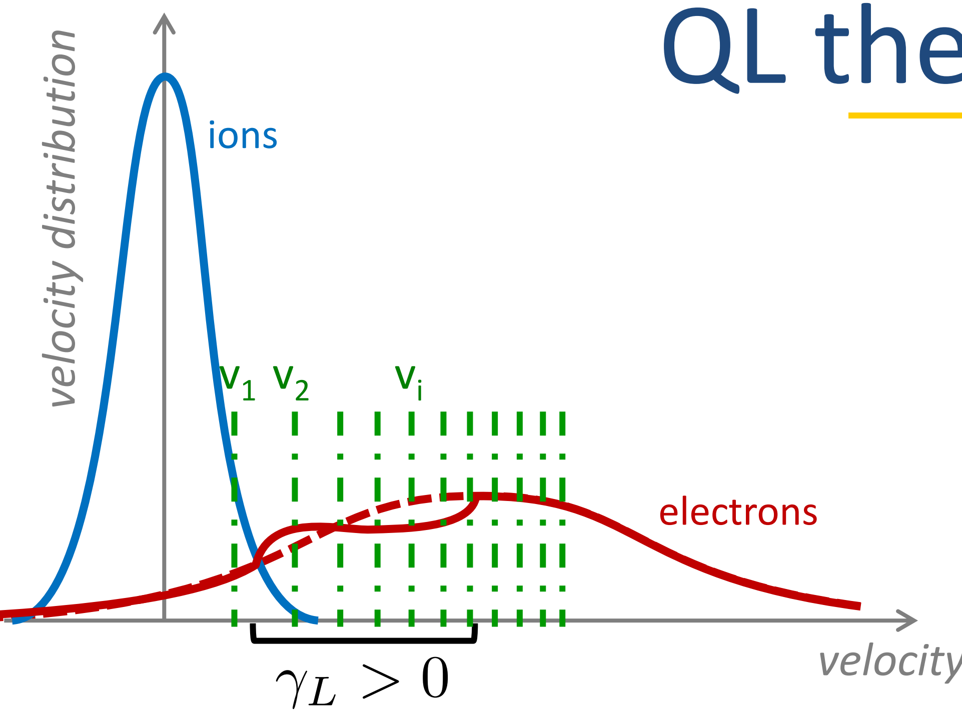


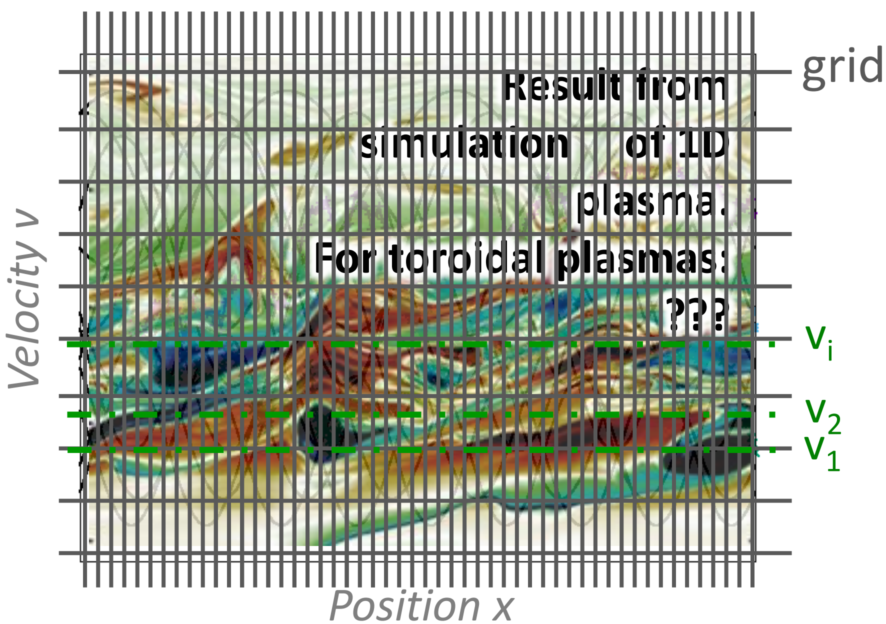


$\Rightarrow$ BGK mode

Bernstein & Green & Kruskal ‘57

QL theory: idea





Linear growth rate

$$\gamma_L \sim \left(\frac{1}{m_i} \frac{\partial f_i}{\partial v} + \frac{1}{m_e} \frac{\partial f_e}{\partial v} \right) \Bigg|_{v=\omega_k/k}$$

Quasilinear theory

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v} \left(D_{\mathrm{QL}} \frac{\partial \tilde{f}}{\partial v} \right)$$

$$D_{\mathrm{QL}} \sim \sum_k |E_k|^2 / k$$

$$\frac{\partial |E_k|}{\partial t} = \gamma_L |E_k|$$

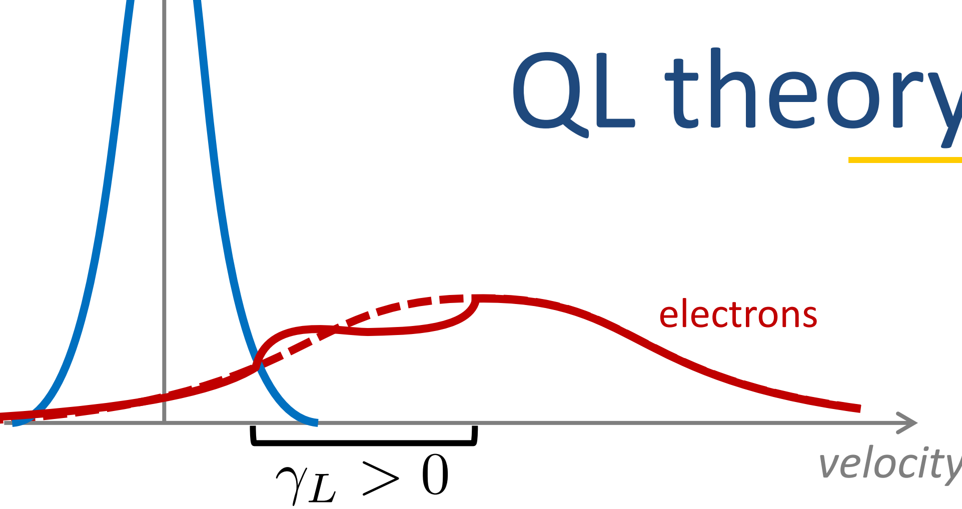
Flattening in the region $\gamma_L > 0$

Vedenov, Velikov, Sagdeev '61

Drummond & Pines '62

Sagdeev & Galeev '69

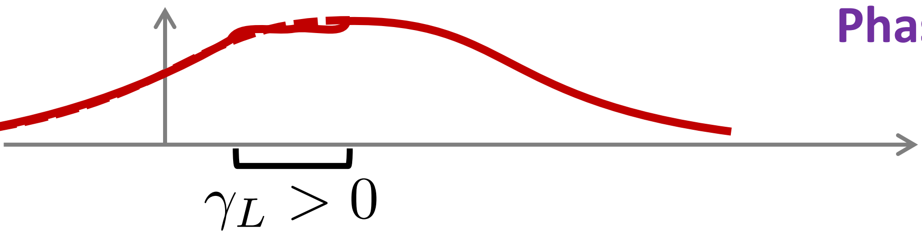
QL theory: limitations



Conventional approach

Flattening in the region $\gamma_L > 0$

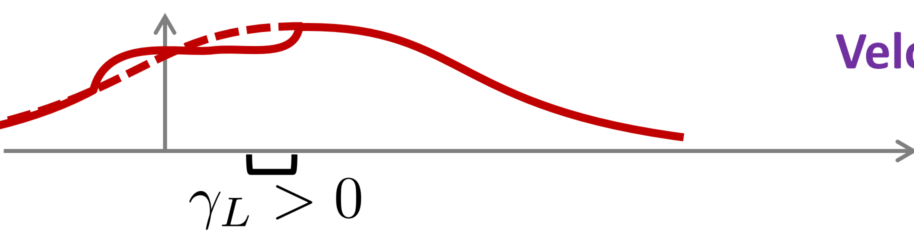
Outside the scope of QL:



Phase-space turbulence

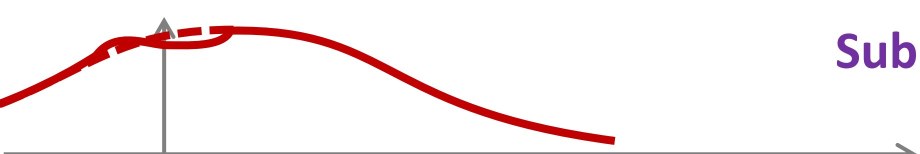
$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v}\left(F_{\mathrm{PS}}\tilde{f} + D_{\mathrm{PS}}\frac{\partial \tilde{f}}{\partial v}\right)$$

$$D_{\mathrm{PS}} \gg D_{\mathrm{QL}}$$



Velocity-space avalanches

Flattening extends to a region $\gamma_L < 0$



Subcritical instabilities

Heating even though $\gamma_L < 0$ everywhere

Small-scale energy-space structures in GK

<u>Kinetics:</u> **6D Phase-space** (3D + 3V)

$\omega << \omega_{gyro}$

<u>Gyrokinetics:</u> **4D Phase-space + 1 parameter**

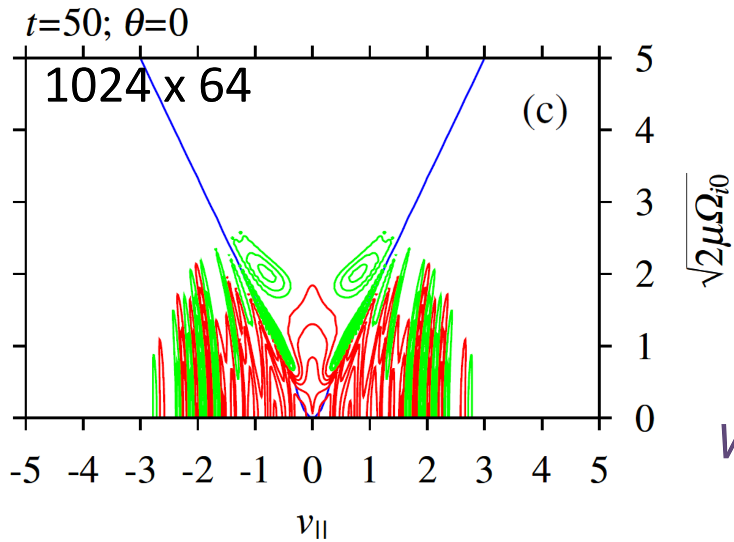
Existence of fine-scale structures in velocity-space

- Not an issue?

  Entropy and energy flux constant between 4 x 4 x 2 and 16 x 16 x 2

  Candy '06

- Still an issue!



Watanabe '06

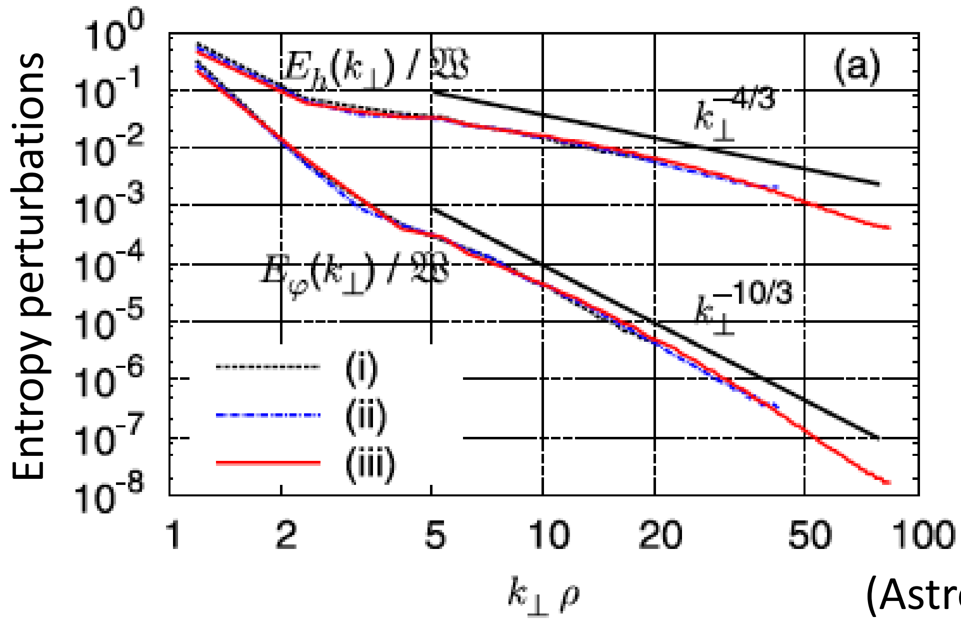
⇒ Existing GK simulations may miss effects of small-scale PS structures

Cascade in velocity-space

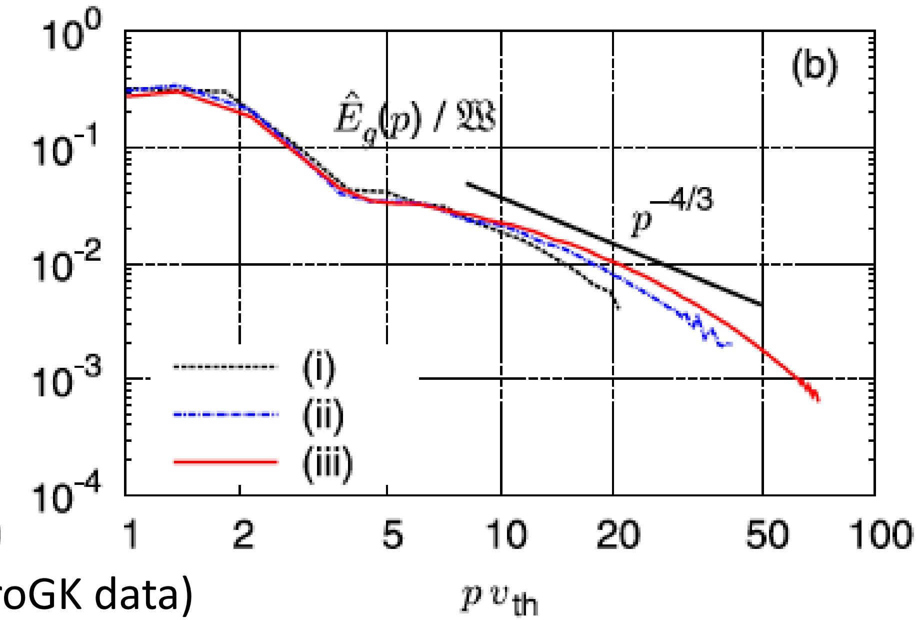
Turbulent cascade of entropy in phase-space

- Driven by nonlinear phase-mixing in strongly turbulent plasmas
- Generation of small-scales in real space and that in velocity space are intertwined.

Schekochihin '08
Tatsuno '09



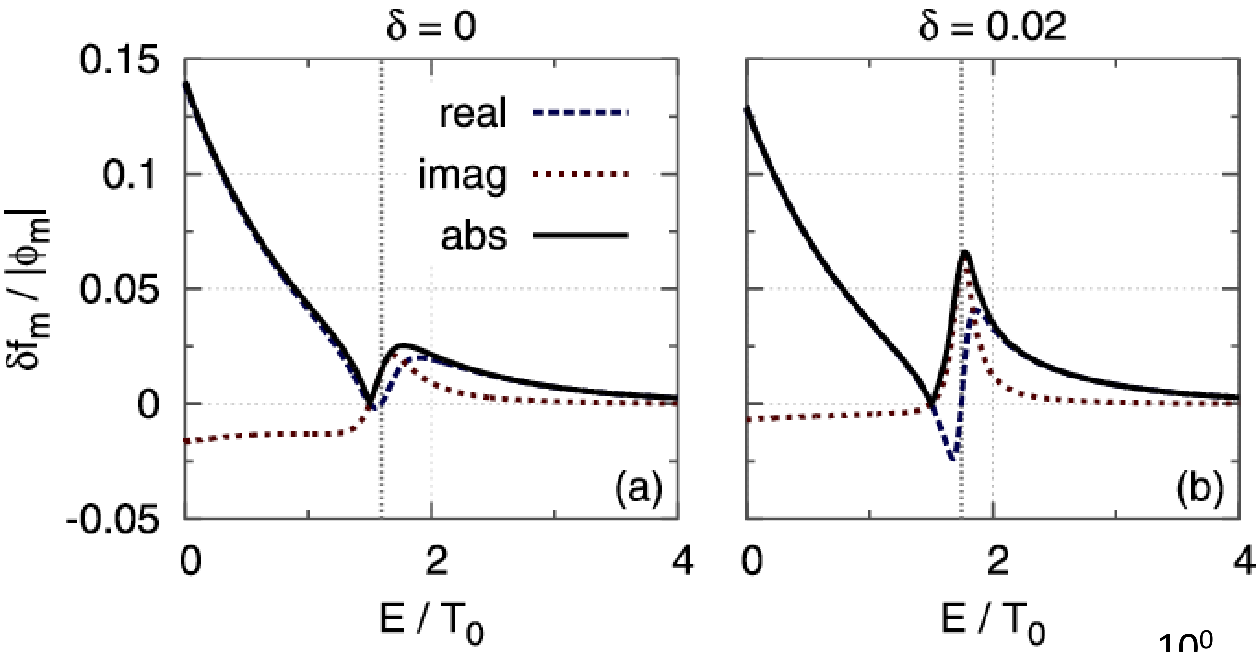
(AstroGK data)



Spectrum in wave number

Spectrum in velocity-space (Hankel spectrum)

Linear analysis



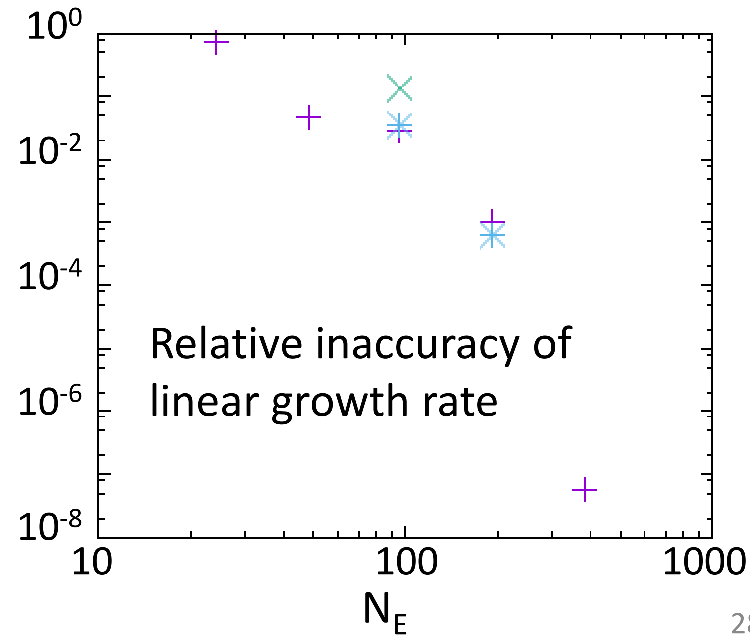
$$\frac{\hat{n}_{e,m}}{n_0} = (1 + \imath m\delta)\frac{e\hat{\phi}_m}{T}$$

$$\delta \approx \epsilon_0^{3/2}\bar{\omega}_{*e}\eta_e/\nu_{ei}$$

Resonant wave-particle interactions can create a narrow structure in energy-space

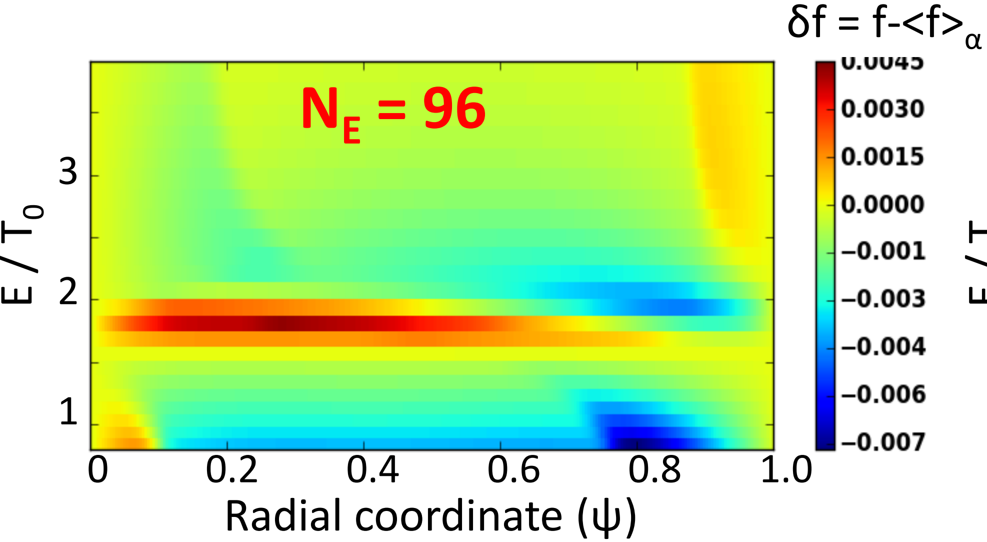
- 10% accuracy requires $N_E \approx 200$
  1% 1000
- Linear growth-rate is not so sensitive (0.1% for $N_E \approx 200$).

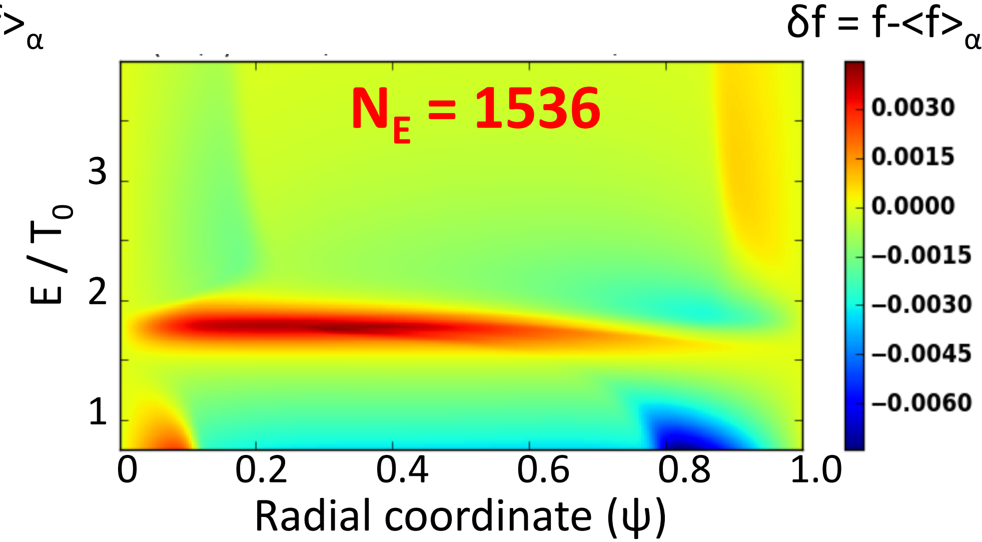
Lesur '17



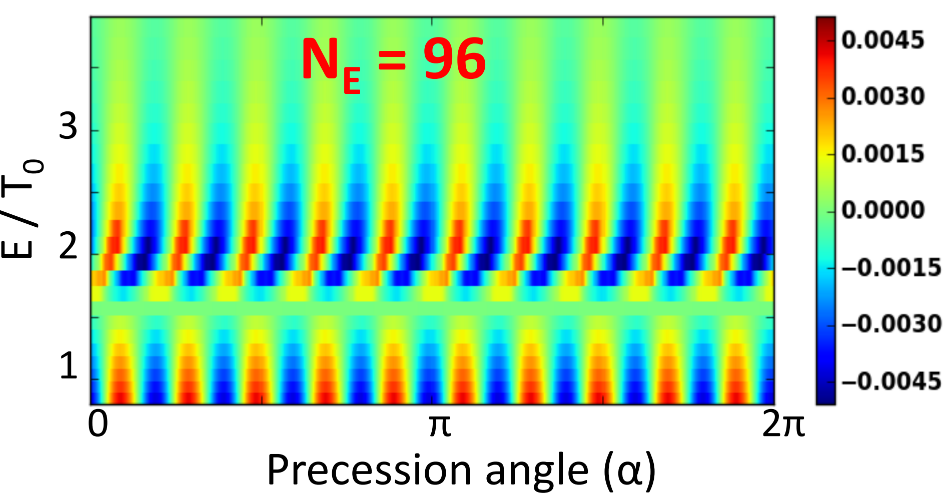
First peak: not-so-fine scales in energy-space

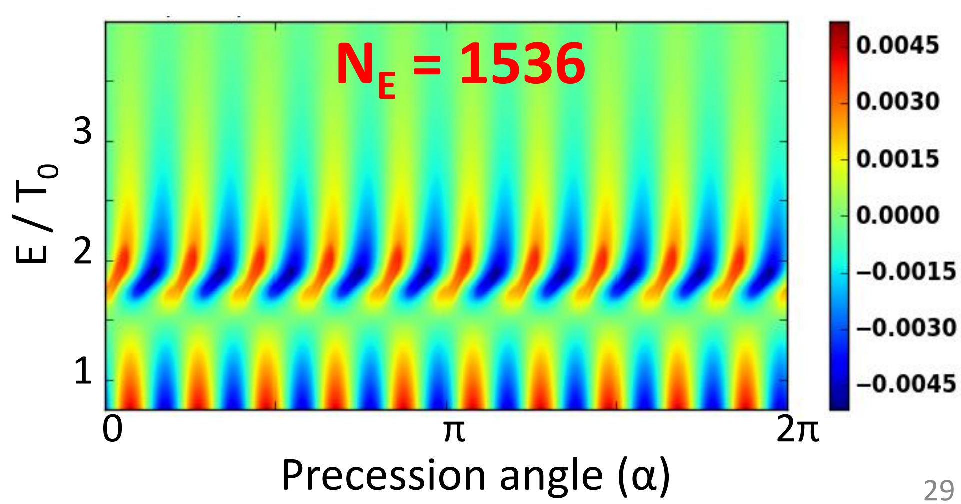
Slice at a given precession angle:





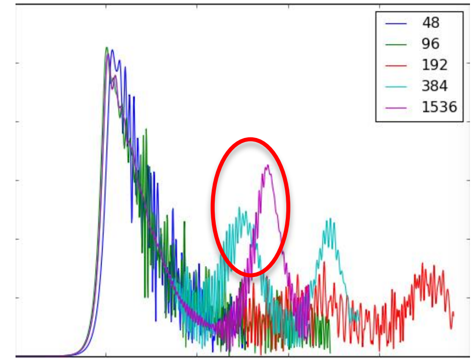
Slice at a given radius:

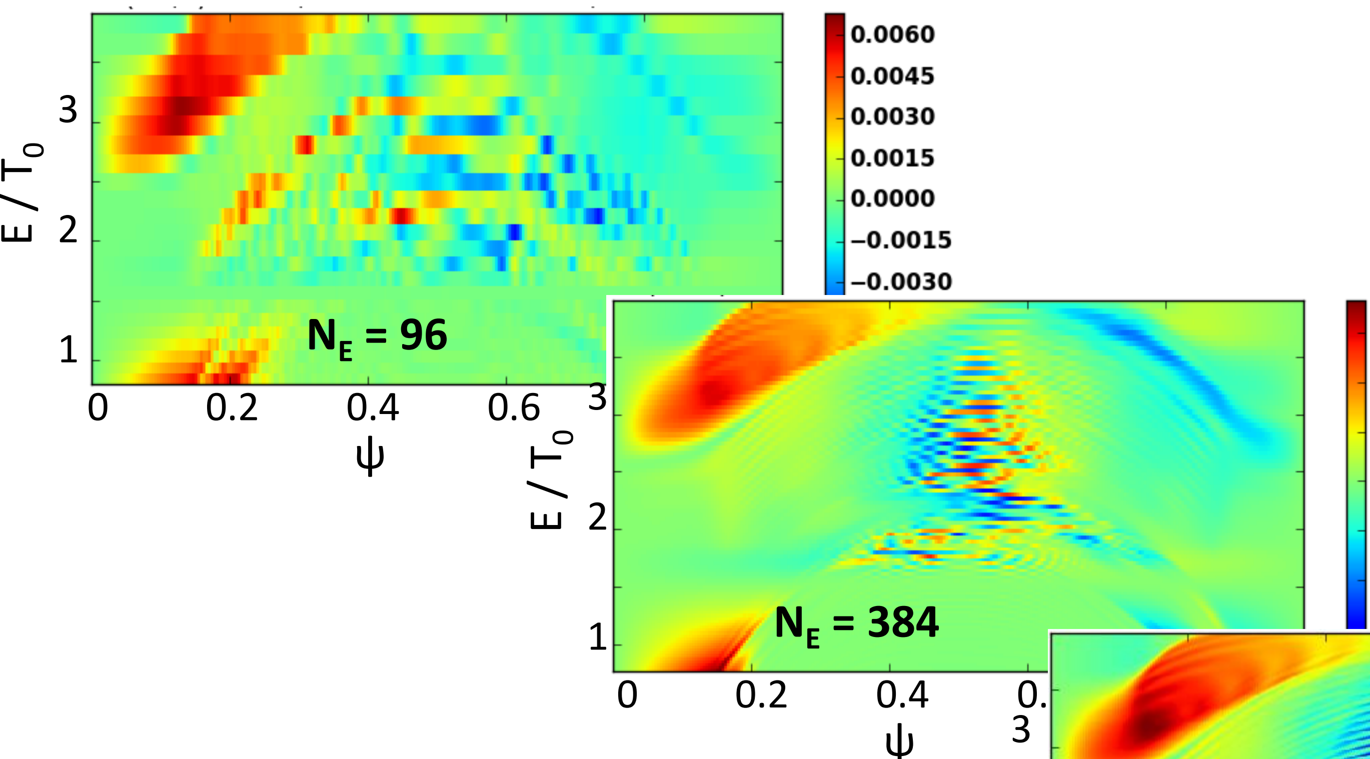


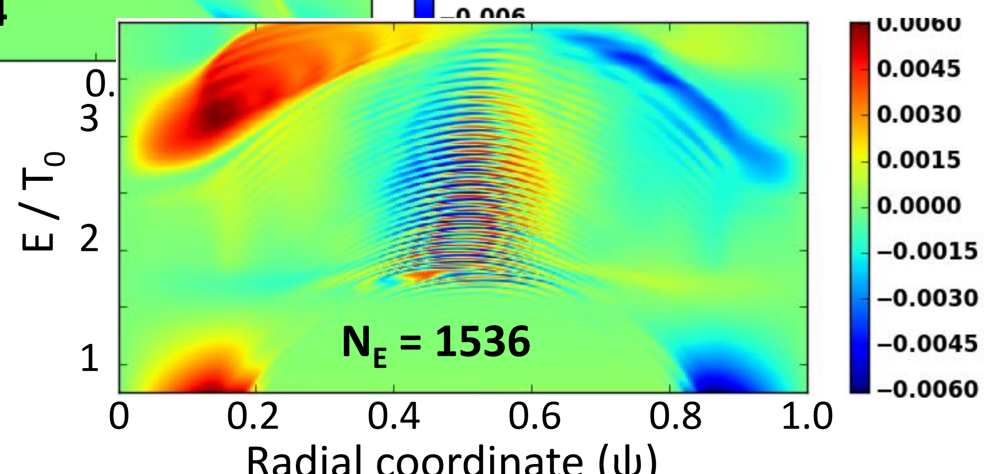


Second peak: slice at a given precession angle

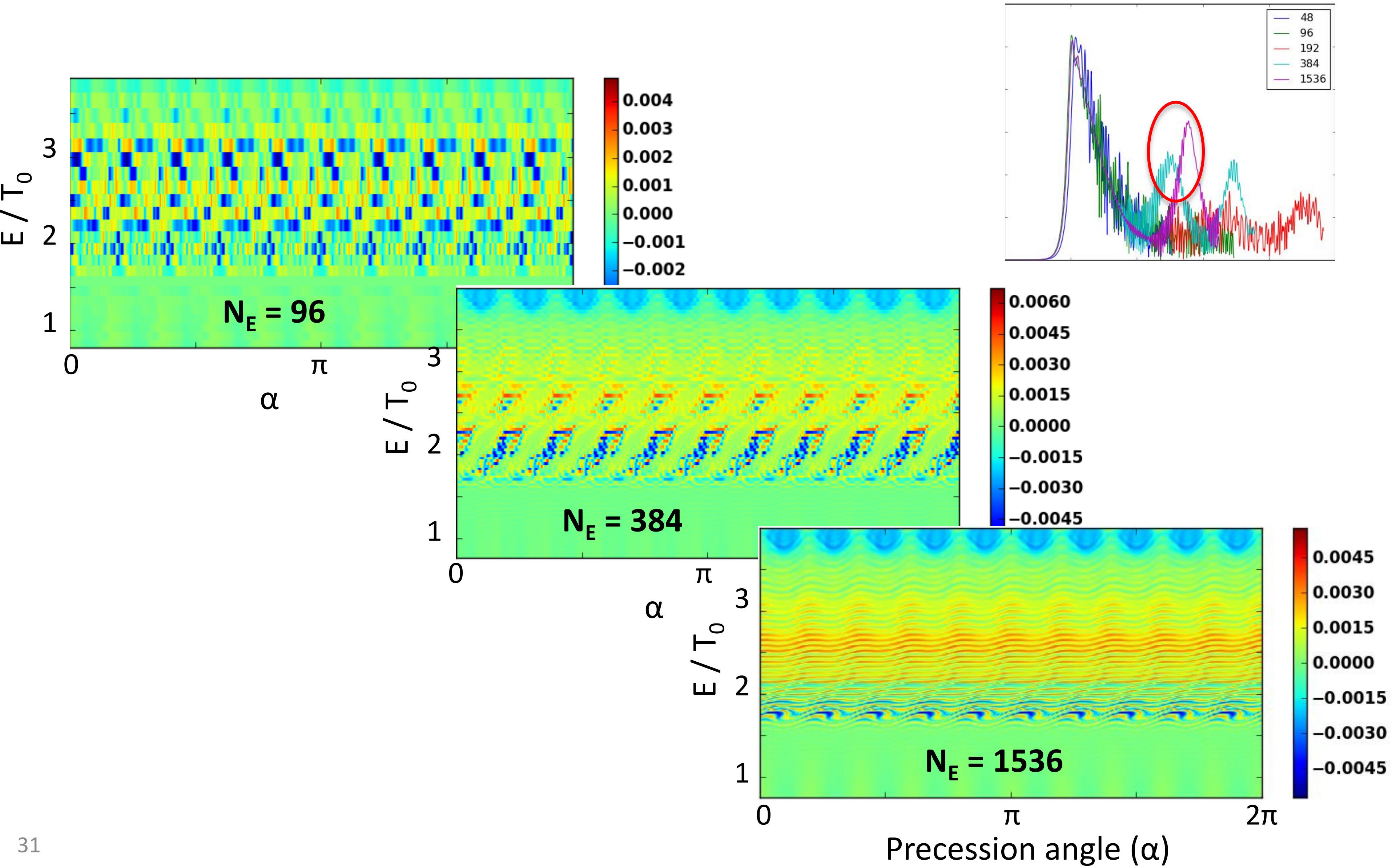
The second peak is associated with the growth of phase-space structures with $\Delta E \ll T_0$



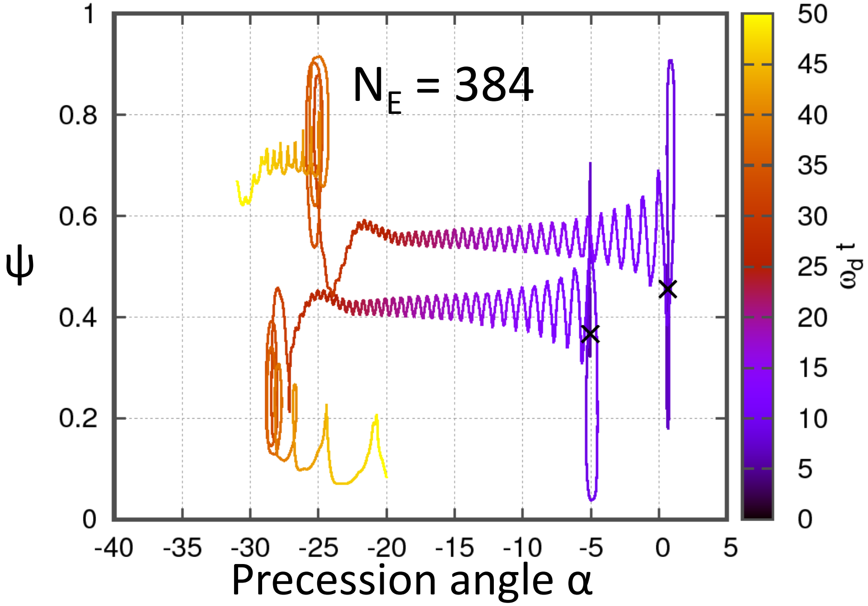


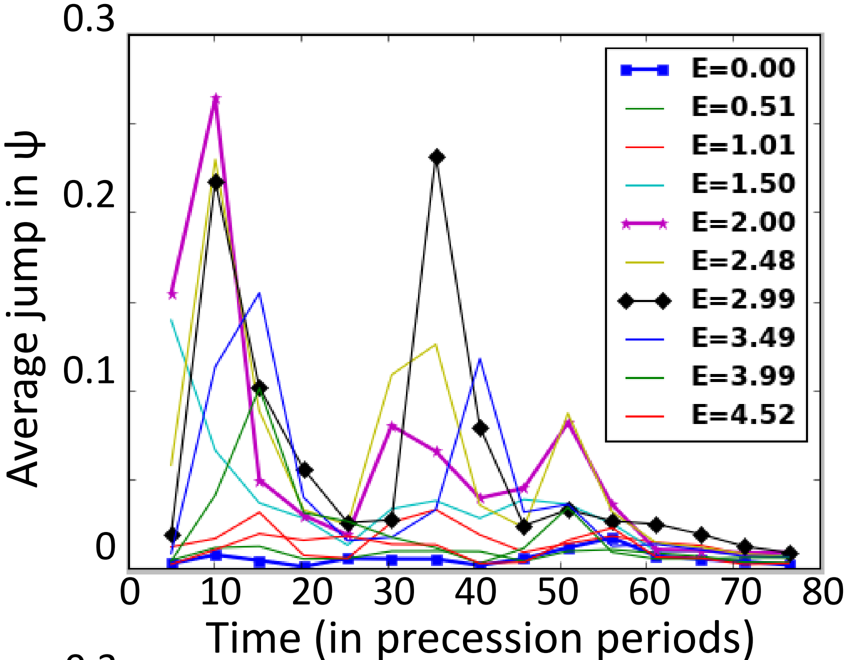


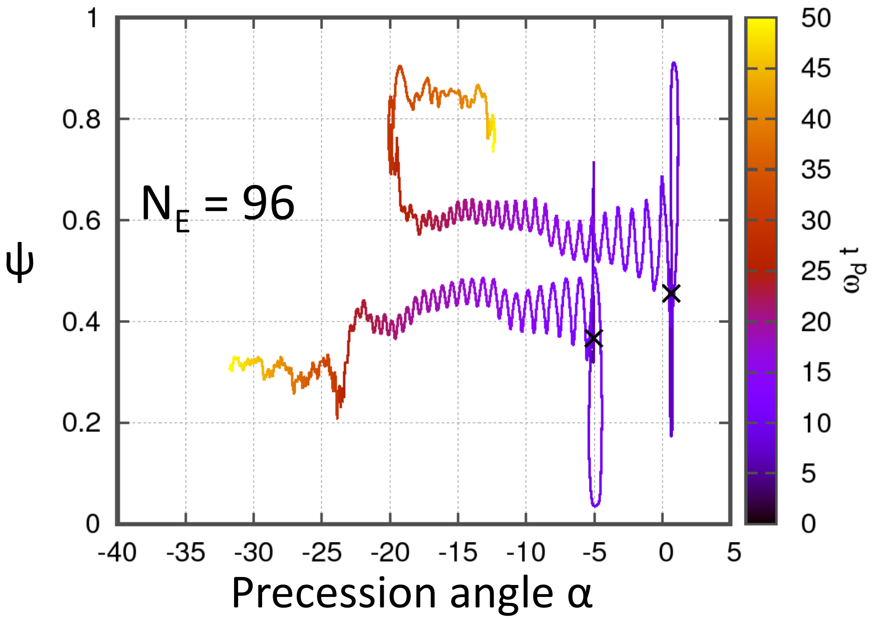
Second peak: slice at a given radius

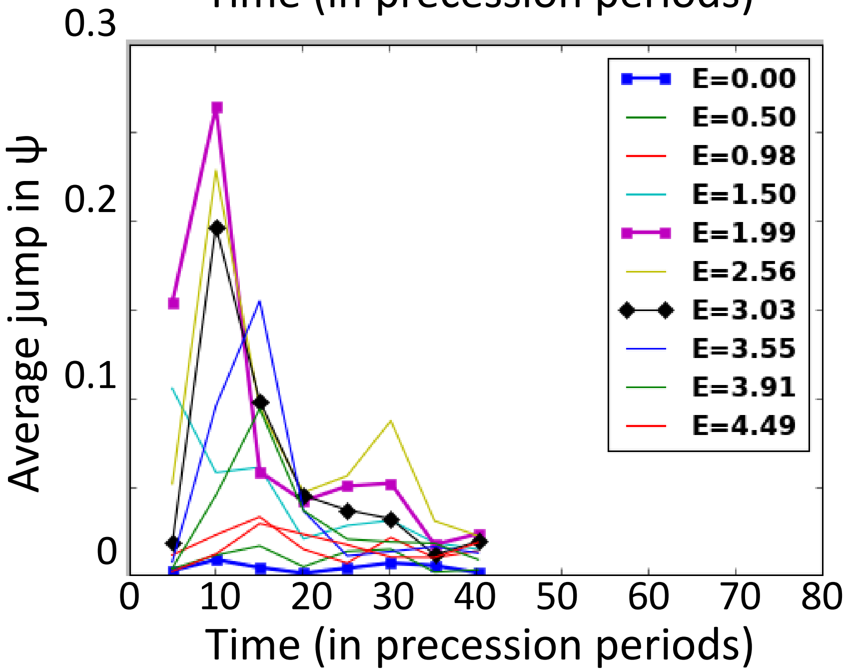


Particle orbits

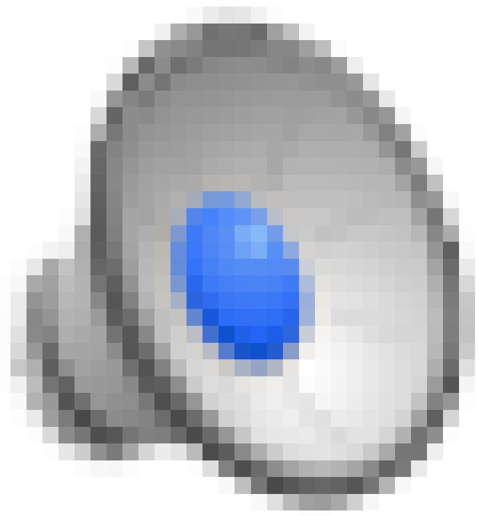




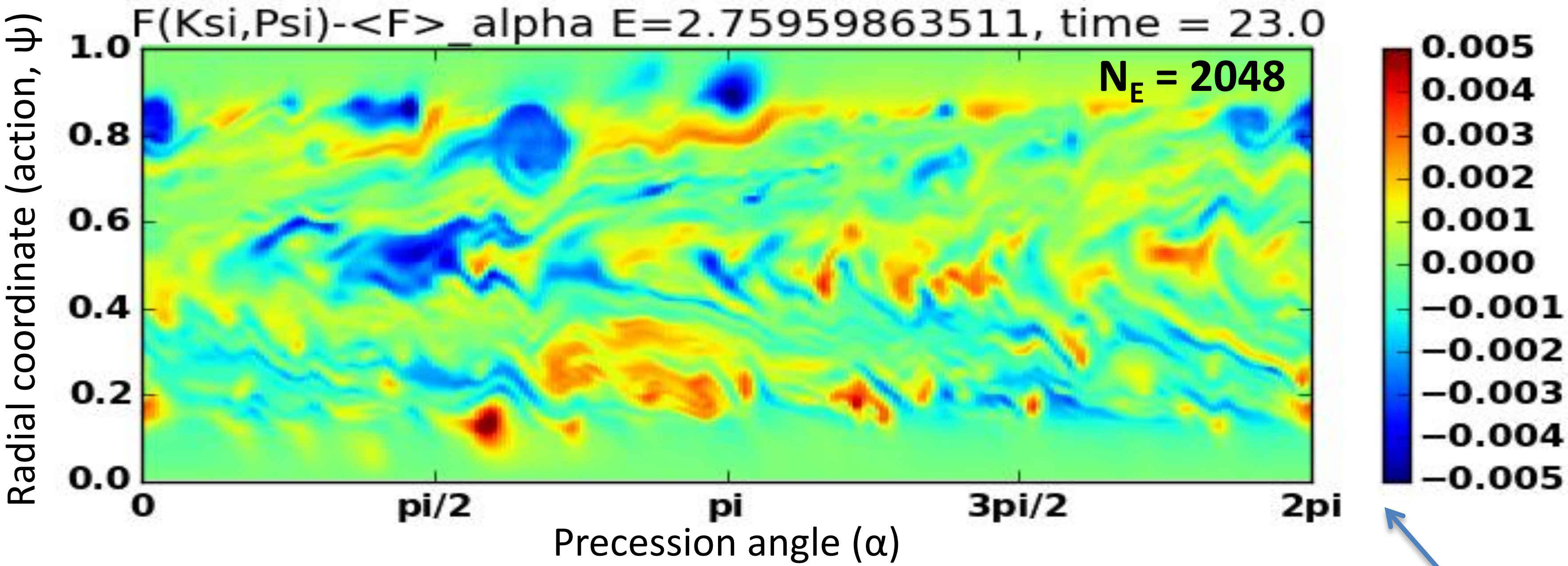


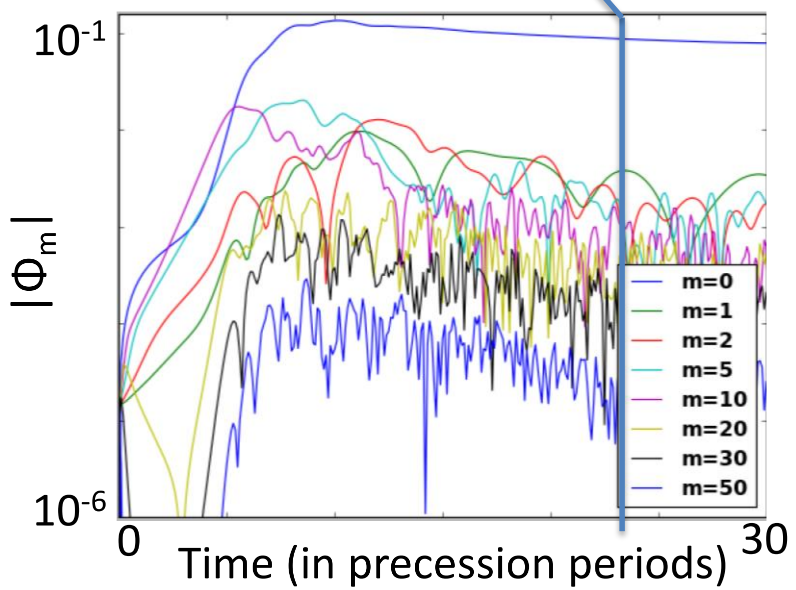


Radially-dependent precession frequency



These structures survive turbulence





Properties

1. Self-organized from seeds born at the resonant energy.
2. Evolved toward higher energies.
3. Localized in radius, size $\Delta r \sim \rho_{ci} \sim$ mm
4. Localized in energy, with $\Delta E \ll T$
5. Lifetime 1 – 10 ms >> drift period