Stability of an island in phase space

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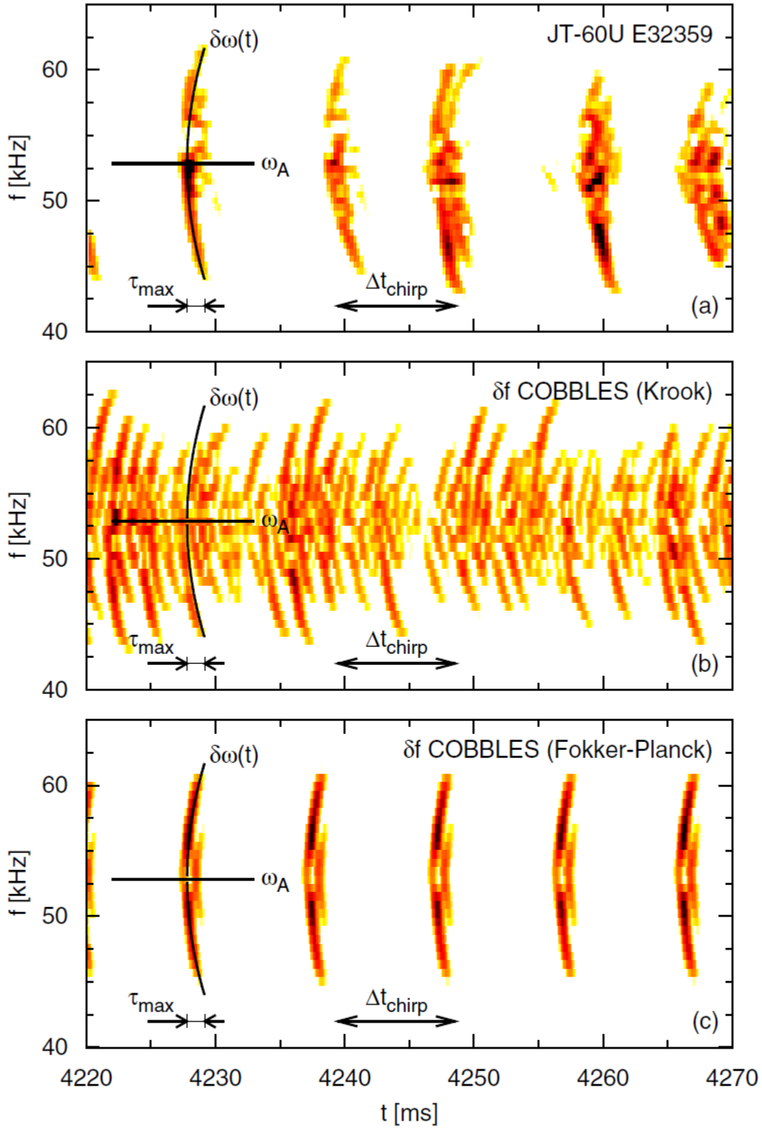
Outline

- The bump-on tail instability.
- Primary wave and non linear evolution.
- Secondary modes and chirping.
- Some further complications.

Motivation

- Alfvén waves driven by energetic particles → transport → potential issue in a fusion plasma.
- Frequency often chirps Wong 99, Lauber 13, Gorelenkov 14, Breizman 11, Chen & Zonca 16
- Conditions for chirping ?



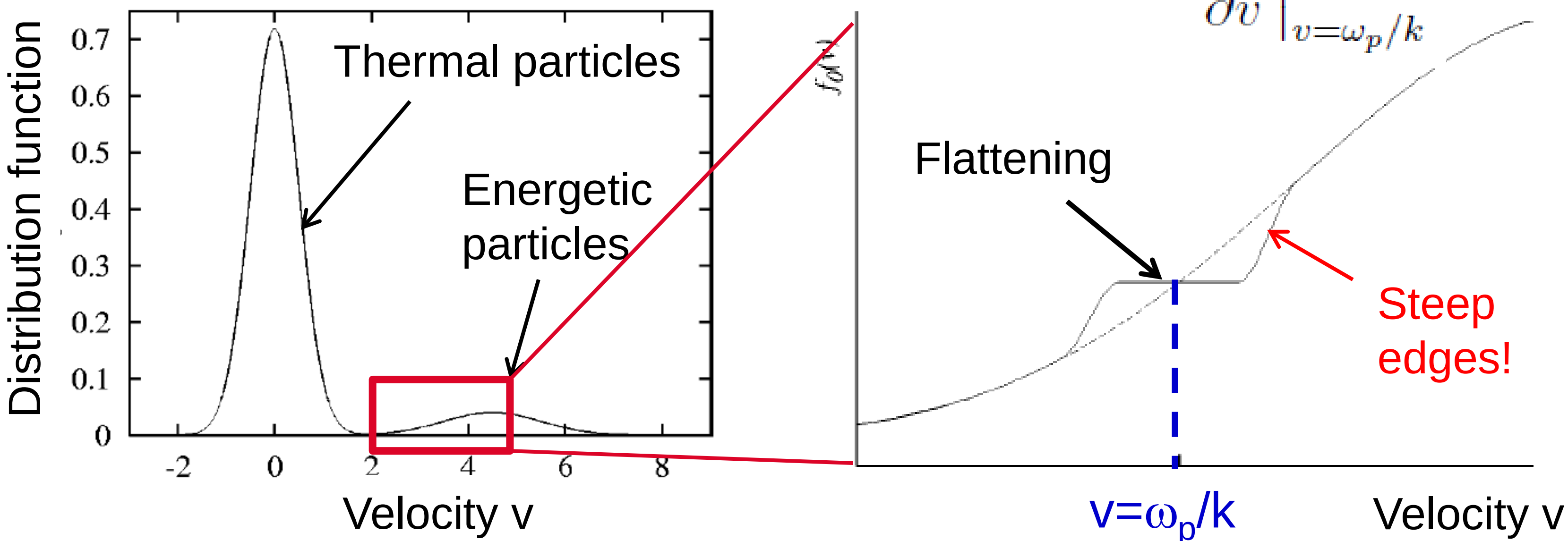
Lesur 10
JT60U/COBBLES

Bump-on tail instability: a prototype of wave particle resonant interaction

- MHD mode + energetic particles in tokamaks $\rightarrow$ bump on tail instability Berk & Breizman 90 : Langmuir wave ($\omega=\omega_p$, k) interacts with an electron warm beam - resonance $\omega_p = k\,v$.

- Instability driven by positive slope of the distribution function $\rightarrow$ transport $\rightarrow$ flattening $\rightarrow$ saturation.

$$\gamma_L \propto \left.\frac{\partial F}{\partial v}\right|_{v=\omega_p/k}$$



Vann 04

Model and numerical solution

- 2D (x,v) Fokker-Planck equation coupled to Maxwell equation

$$\frac{\partial F}{\partial t} + v\frac{\partial F}{\partial x} - \frac{eE}{m}\frac{\partial F}{\partial v} = C(F)$$

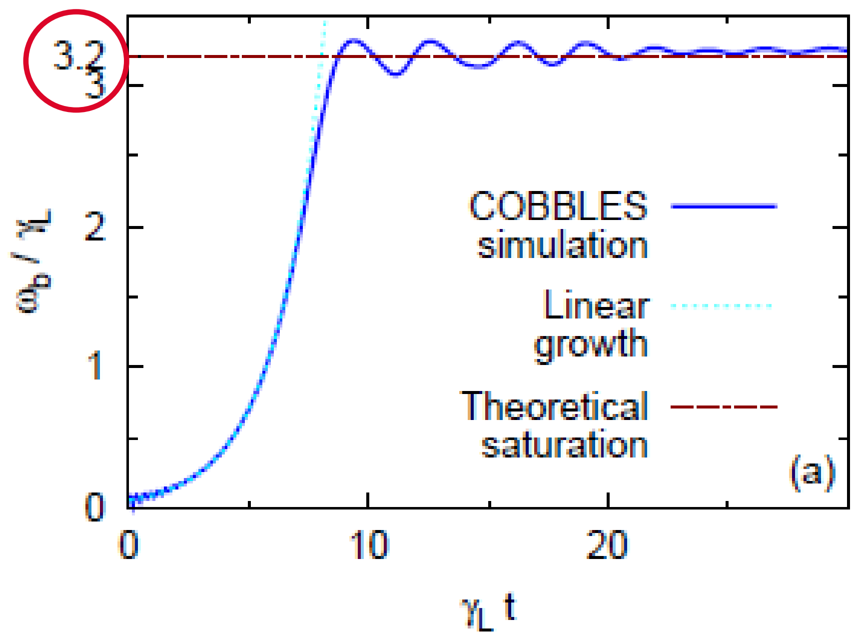
$$C(F) = \frac{\partial}{\partial v}\left(D(v)\frac{\partial F}{\partial v} + \nu_f(v)F\right)$$

Collisional scattering

Collisional drag

$$\frac{\partial E}{\partial t} = \frac{e}{\epsilon_0}\int dvv\,(F - F_0) - 2\gamma_d E$$

Wave damping



f periodic in x, null at v → ∞

- COBBLES code – Lesur 10

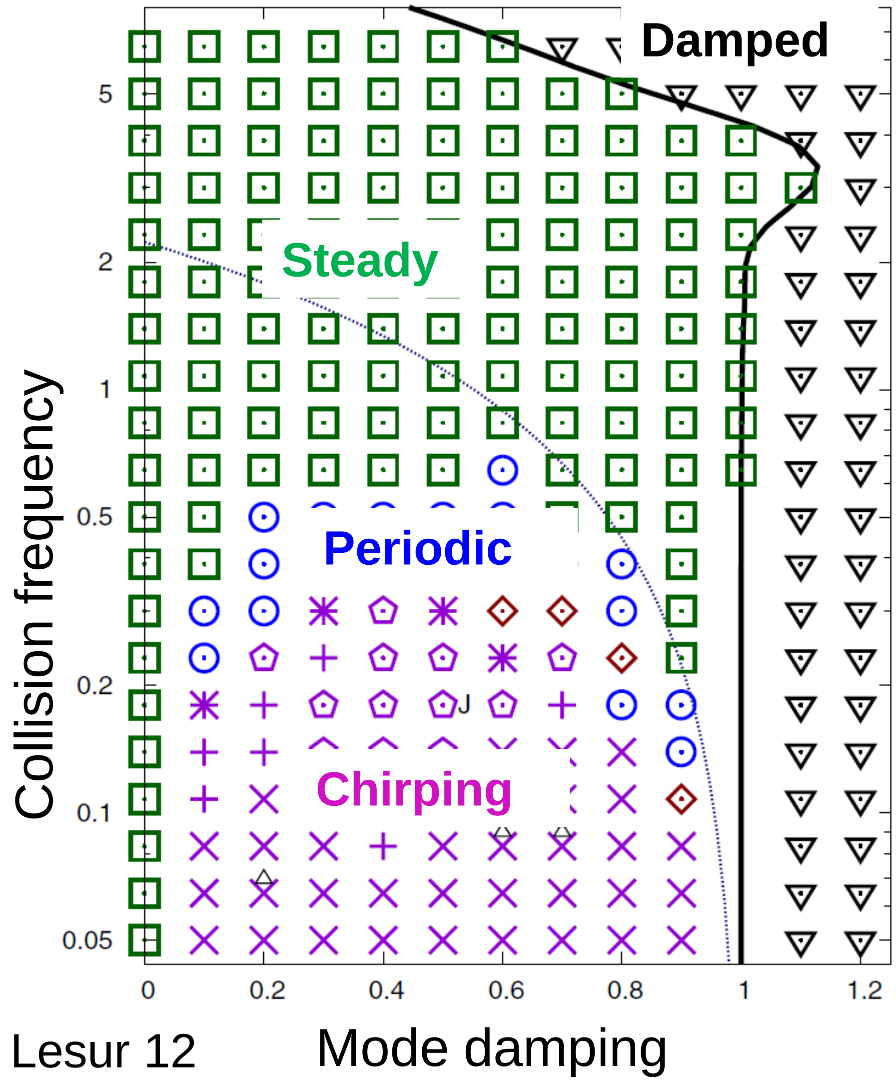
Oscillations O'Neil 65, Mazitov 65

Saturation Fried 71

See OV Galeev & Sagdeev 73

Dynamics is unsteady at low dissipation

- Nonlinear dynamics depends on dissipation Vann 2003, Lesur 2012
- Phase diagram in parameter space (ν_{coll},γ_d)
- Chirping occurs at low dissipation Berk & Breizman 90



Lesur 12

An island builds up in the phase space

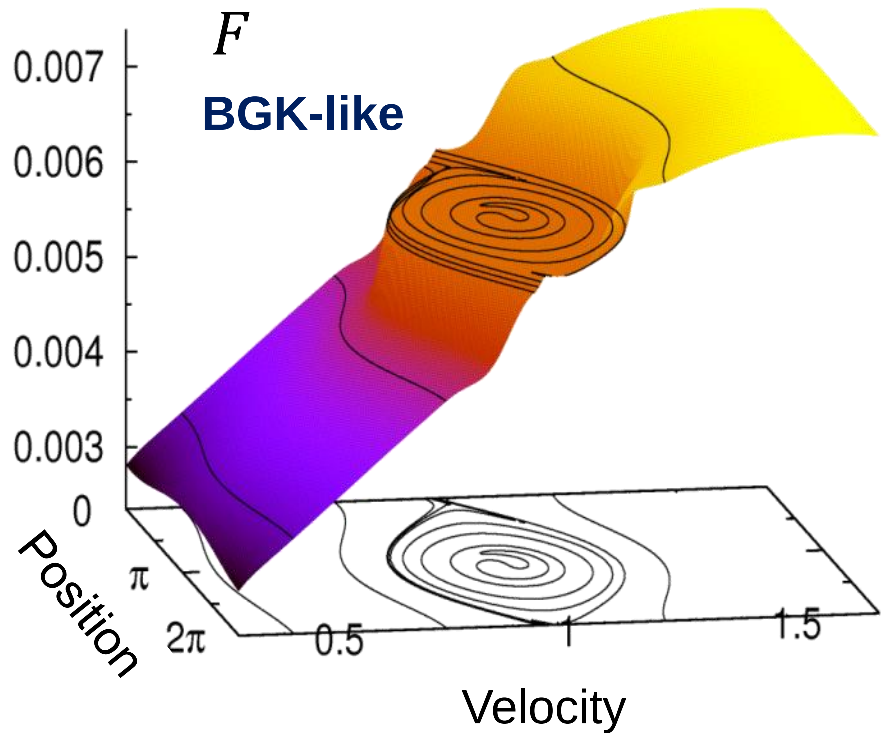
- Lines of constant energy - wave frame $\xi = k_0 x - \omega_p t$, $v \rightarrow v - \omega_p/k$

$$H_0 = \frac{1}{2} m v^2 - e\phi_0 \cos\xi$$

$\rightarrow$ island in the phase space (x,v) near the resonance.

- Distribution $F(H_0, \xi)$ solution of a Fokker-Planck equation

$$kv \frac{\partial F}{\partial \xi} = C(F)$$



Island half-width

$$\Delta v = 2\sqrt{\frac{e\phi_0}{m}}$$

Steady final state is a flattish distribution in the island

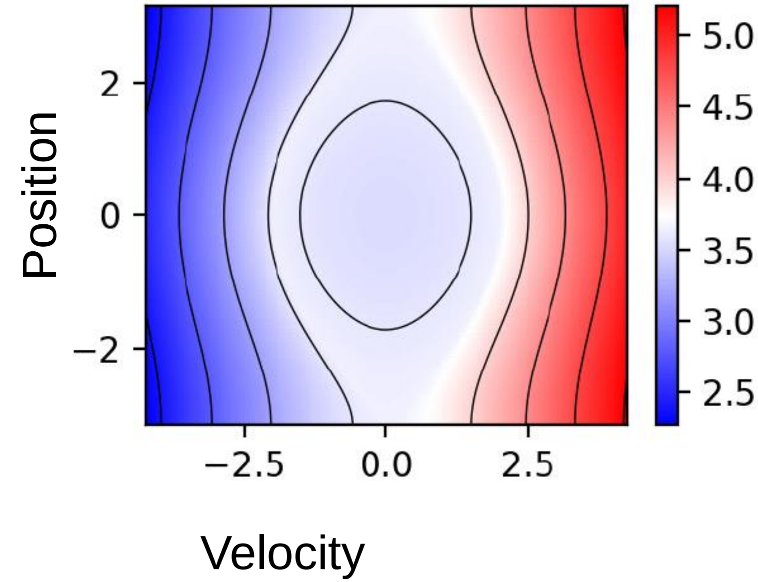
- Analytical solution for small diffusion, no drag Zakharov-Karpman 62

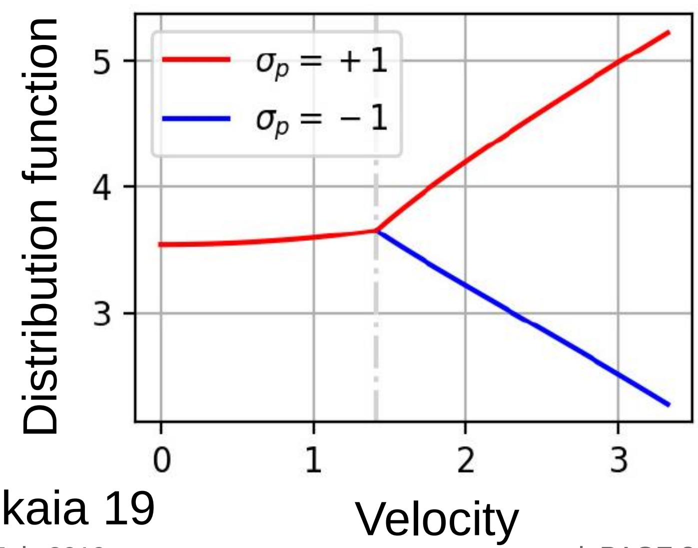
$$F = F_0(H_0) + \epsilon_D F_1(H_0, \xi)$$

$$\oint d\xi \frac{C(F_0)}{\sqrt{H_0 - e\phi\cos\xi}} = 0$$

- Numerical solution of Fokker-Planck equation benchmarked against analytical solution and COBBLES code.

- Slope depends on dissipation.





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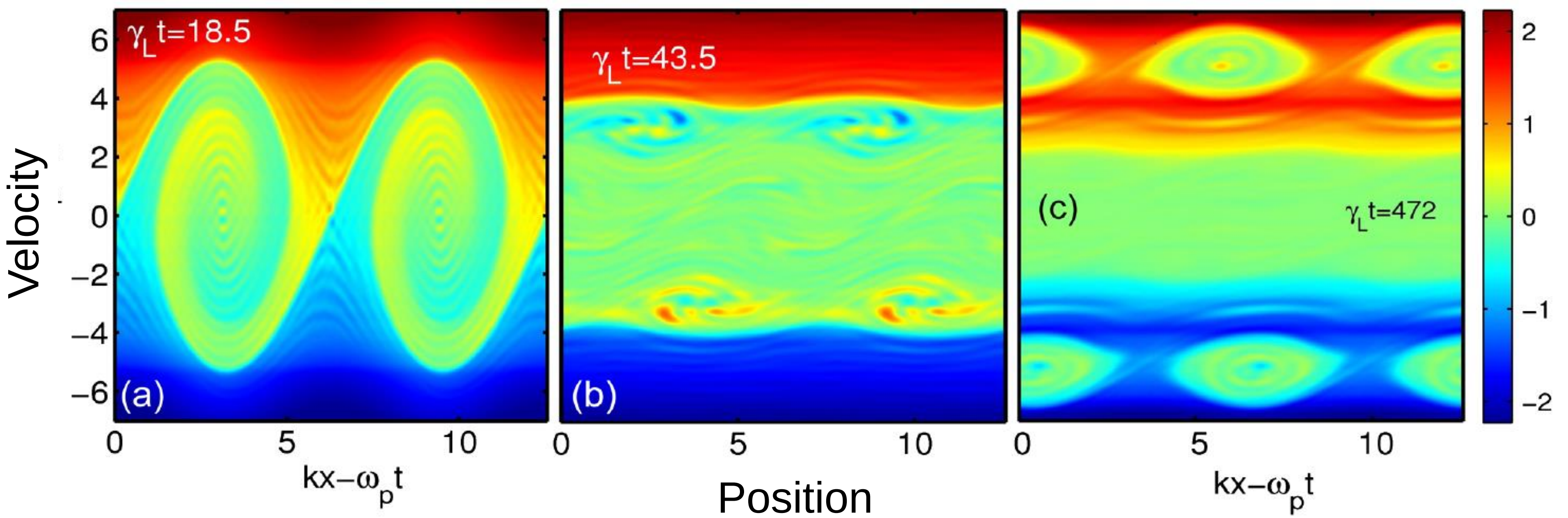
Clump/hole formation and chirping

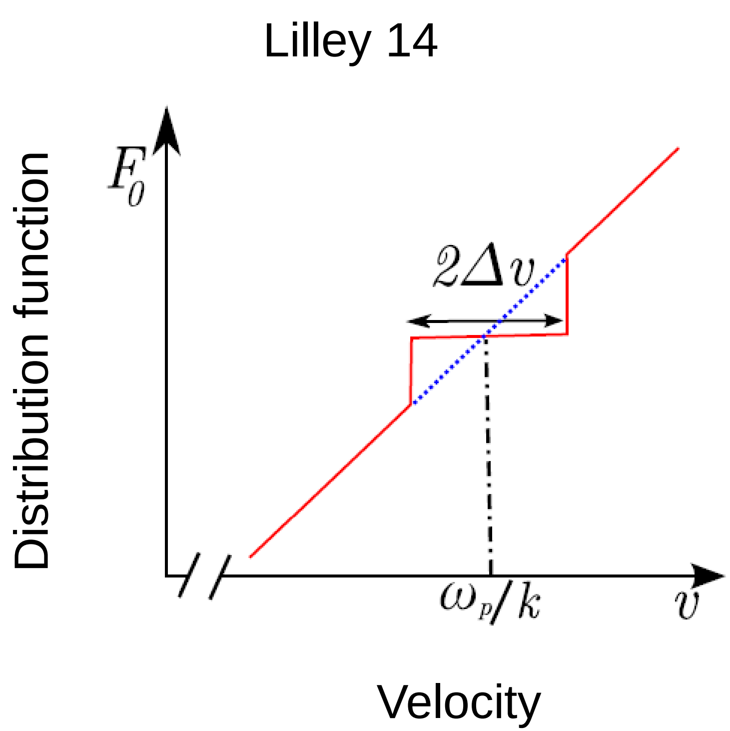
- At low dissipation, island splits in a pair of clump/hole Berk, Breizman & Petviashvili 97
- Clump/hole part and drift → chirping $\omega \sim \gamma_L (\gamma_d t)^{1/2}$ Berk et al. 99
- Produces a transport of energetic particles.

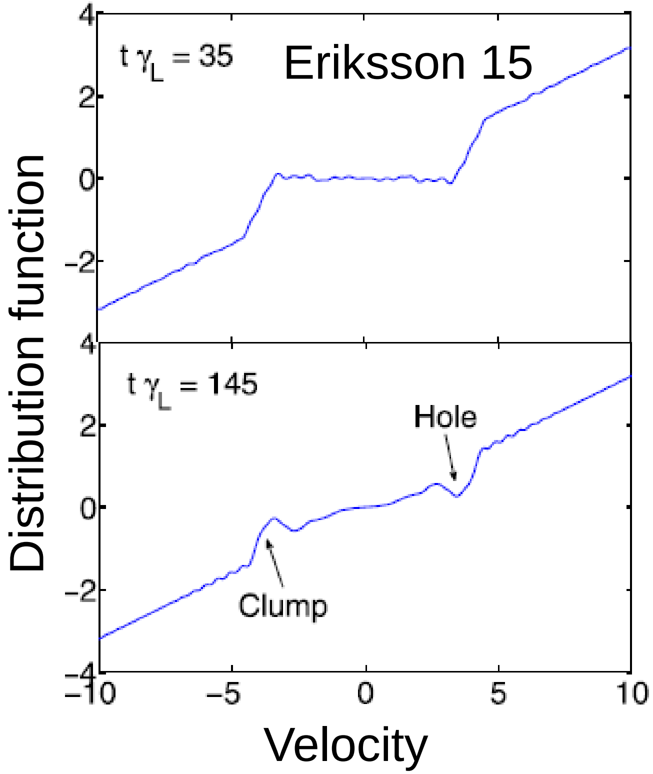
Distribution function Lilley 14



Clump/hole detachment likely due to the onset of secondary instabilities

- Berk-Breizman solution is asymptotic. How does it start?
- Attributed to a secondary instability Lilley 14, Eriksson 15
- Collisionless primary structure is an annulus, not an island.





What is going on at larger dissipation?

- At high dissipation: quasi-steady island forms
- Stable against secondary modes or not?

$\rightarrow$ revisit stability of primary mode

Strategy to find secondary instabilities

- Primary wave $(\omega_p, k_0) \rightarrow$ new equilibrium $F_0(H_0)$ – reflects the island shape.
- Linearly perturbed by a secondary mode with wave number $k = \ell k_0$ and pulsation $\omega = \ell\omega_p + \delta\omega$

$$\delta\phi(\xi) = \phi_1 \exp\left\{i\left(\ell\xi - \delta\omega t\right)\right\} + c.c.$$

$$F(H_0, \xi) = F_0(H_0) + \delta F(H_0, \xi, t)$$

- Solve Vlasov + Poisson equations

Dispersion relation of secondary modes

- A dispersion relation D(δω, ℓ=k/k$_0$)=0 is derived Dudkovskaia 19

Langmuir wave

Density fast particles

$$D\left(\delta\omega,l\right)=-l^2+l^2\sum_{j=e,i}\left(\frac{\omega_{pj}^2}{\omega^2}+2i\frac{\omega\gamma_j}{\omega_{pj}^2}\right)+\sum_{j=fe,fi}\omega_{pj}^2\sum_{\sigma_p}\int_{-\omega_b^2}^{+\infty}\frac{dH_0}{\Omega_b}\frac{\partial f_{0,j}}{\partial H_0}+$$

$$2\pi i\sum_{j=fe,fi}\omega_{pj}^2\sum_{\sigma_p}\int_{-\omega_b^2}^{+\infty}\frac{dH_0}{\Omega_b}\frac{\delta\omega}{|\Omega_b|}\frac{\partial f_{0,j}}{\partial H_0}\int_{-\pi}^{\pi}\frac{d\alpha}{2\pi}e^{-i\left(l\xi-\frac{\delta\omega}{\Omega_b}\alpha\right)}\times$$

$$\times\left\{\int_{-\infty}^{+\infty}\frac{d\alpha'}{2\pi}e^{i\left(l\xi'-\frac{\delta\omega}{\Omega_b}\alpha'\right)}\cdot\Theta\left[\sigma_p\left(\alpha-\alpha'\right)\right]+C\left(\sigma_p\right)\right\}.$$

bounce pulsation

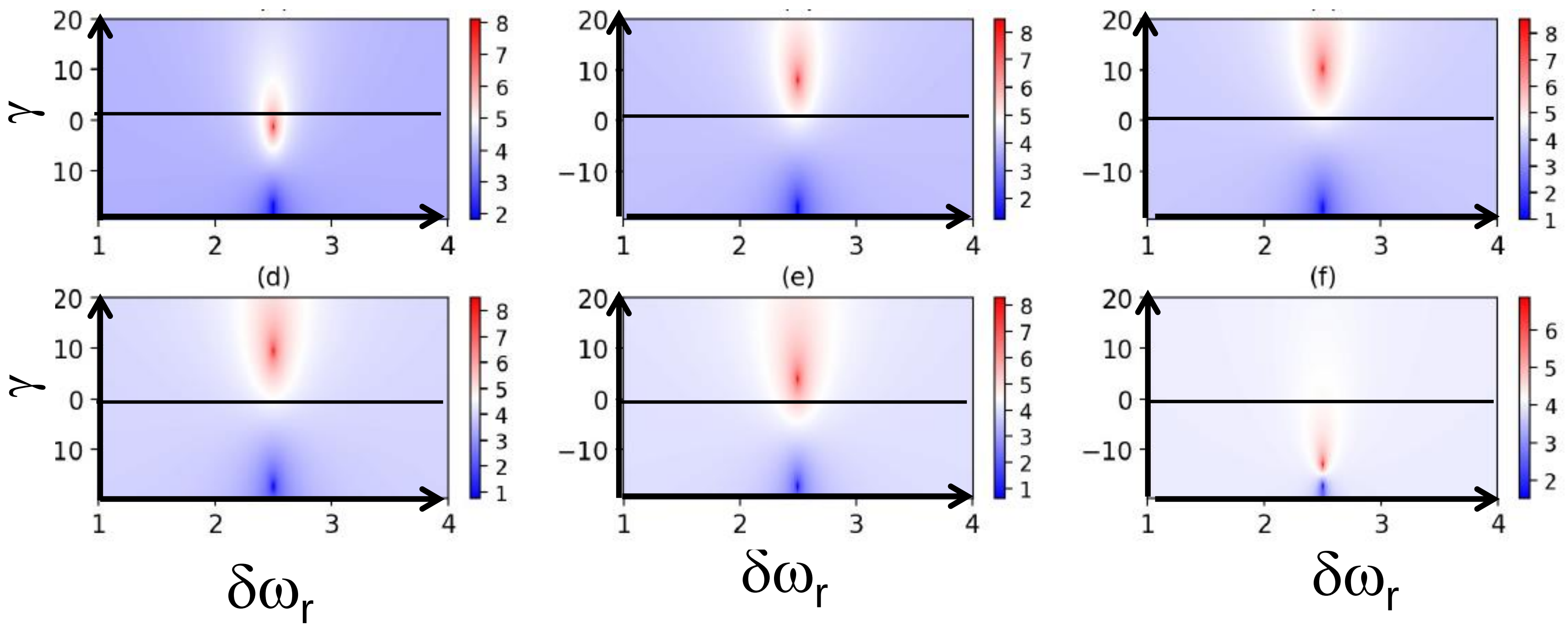
Slope fast particle distribution function

- D(ω, ℓ=k/k$_0$)=0 solved numerically

Secondary instabilities grow for a well defined range of island widths

- $1/|D(\omega, \ell=k/k_0)|$ vs (ω_r, γ): maximum=instability if $\gamma>0$
- Unstable within a range of island width Δv

Zeros of $D(\omega, \ell=k/k_0)$ for growing island width



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Agrees with numerical simulations

Simulations with the COBBLES code:

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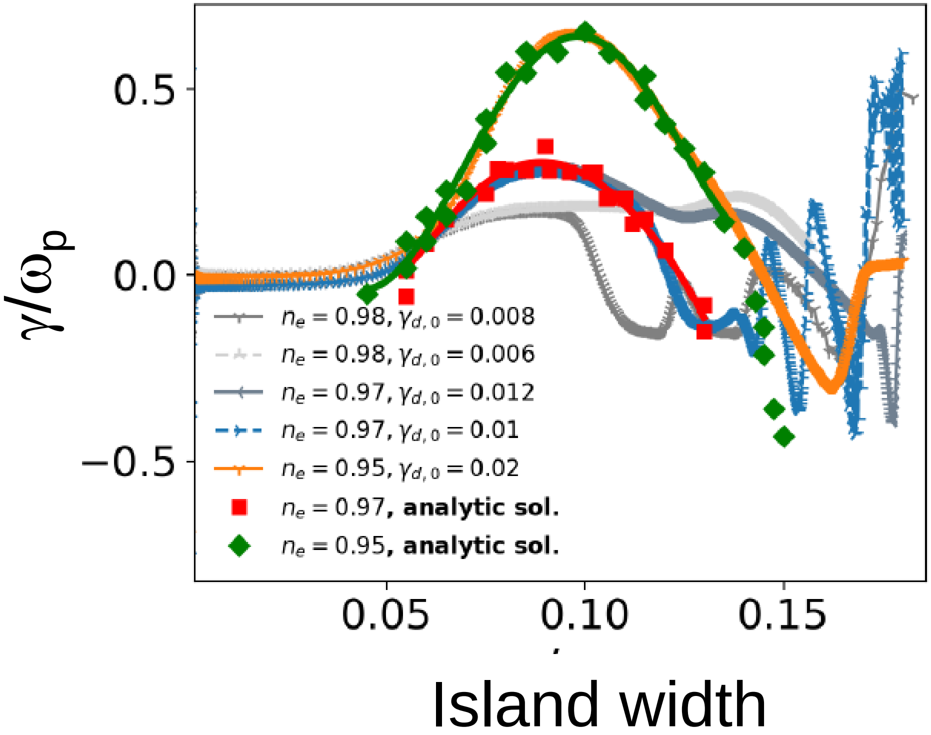
1) starts with a unique primary mode $\rightarrow$ saturates via island formation.

2) onset of linearly stable secondary modes.

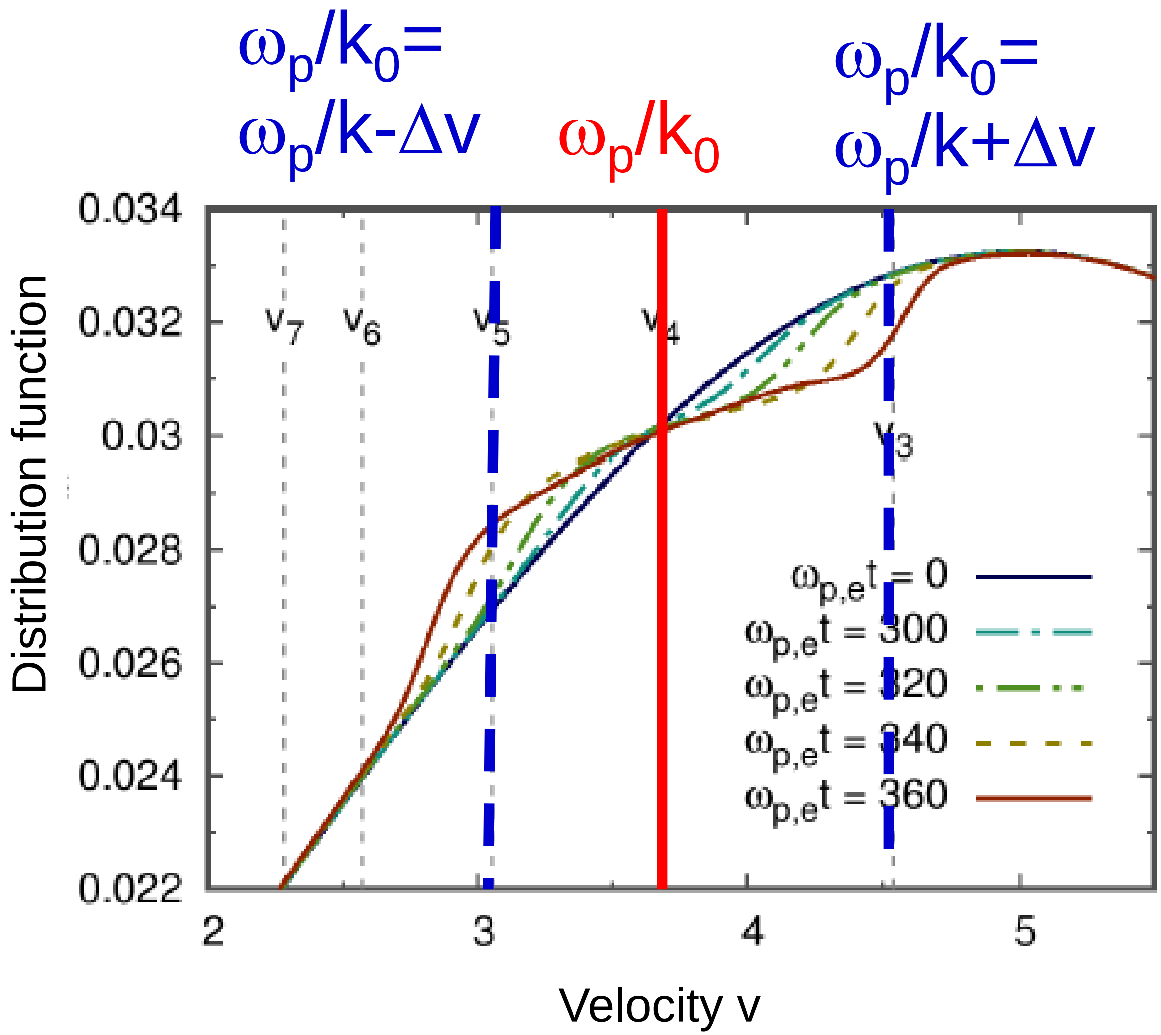
3) growth in accordance with analytic dispersion solution.



Slope steepening must match a secondary resonance

- Physical reason : unstable when accessible resonant phase velocity matches the island edges:

$$\frac{\omega_p}{k} = \frac{\omega_p}{k_0} \pm \Delta v$$

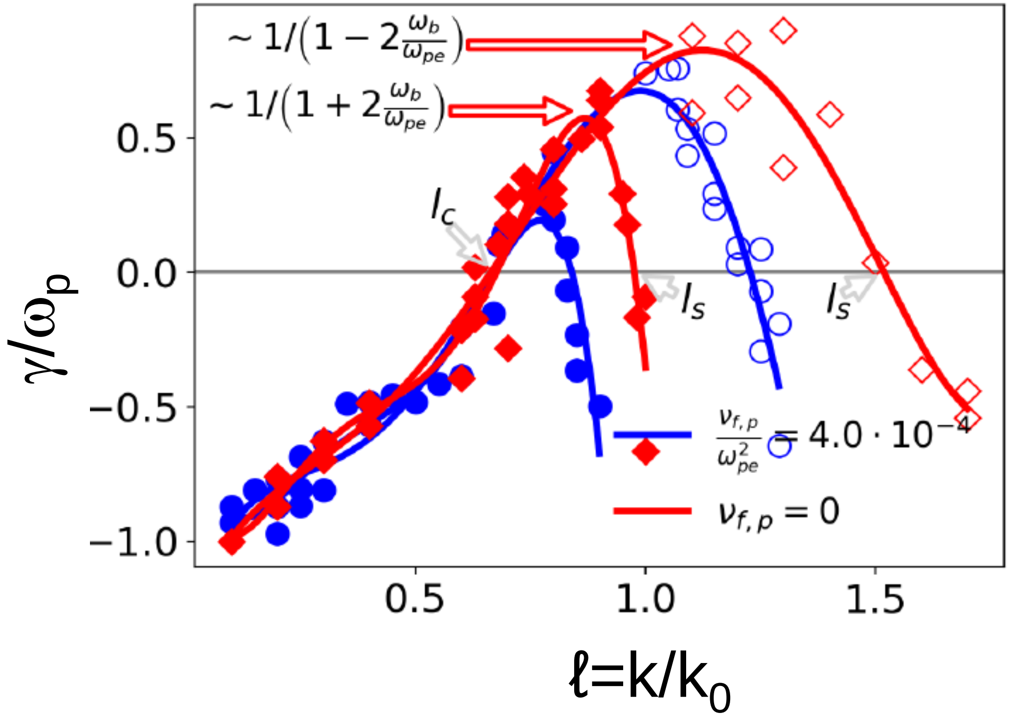


A limited range of wavenumbers is unstable

- Secondary instability when

$$\ell_c < \ell = k/k_0 < \ell_s$$

- Stable if accessible ℓ are all stable $\rightarrow$ chirping does not occur when dissipation is large enough.



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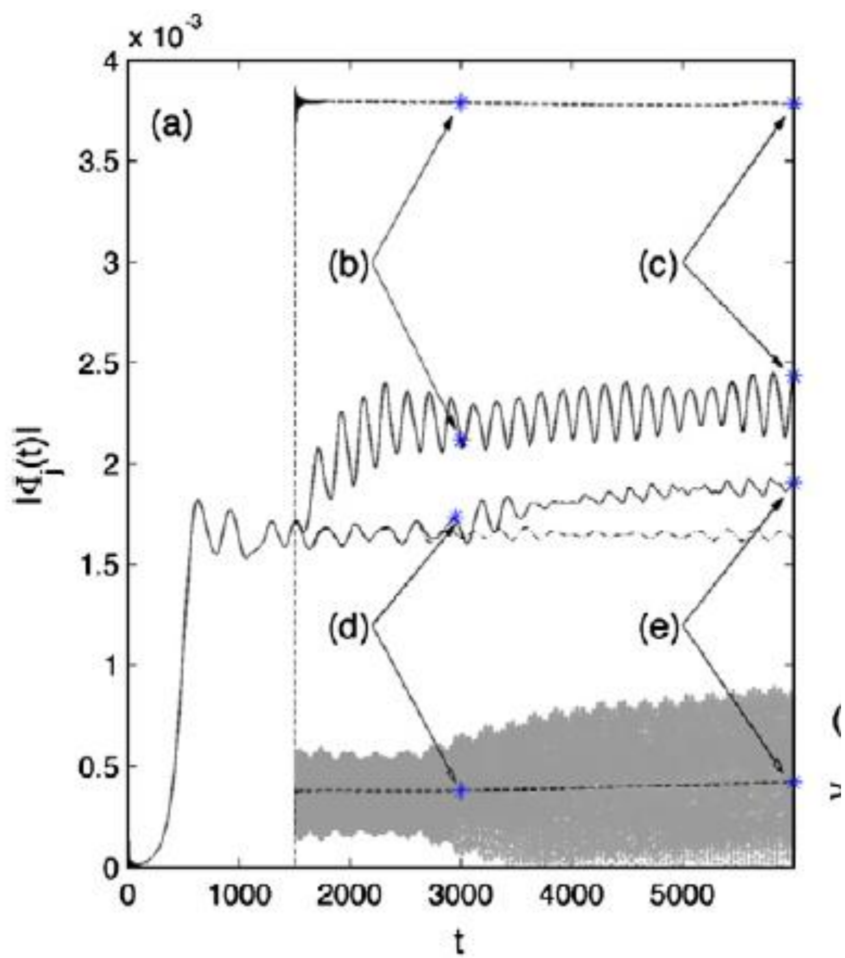
An aside: stability of BGK modes

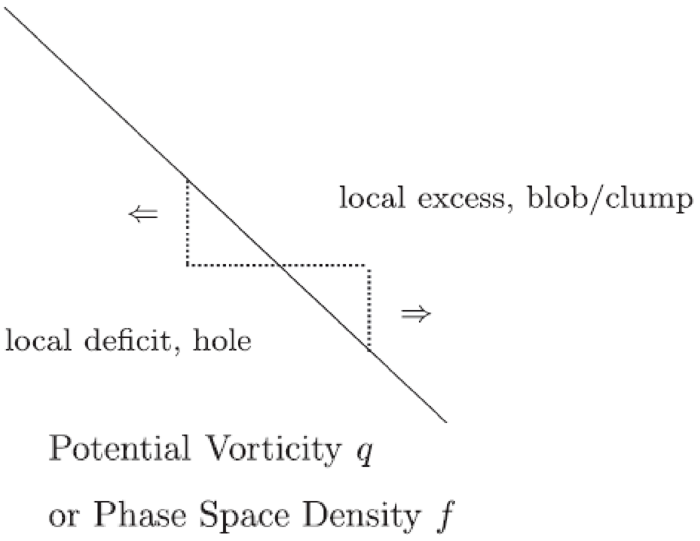
- Bernstein Greene Kruskal 57= Steady non linear solution of FP/Poisson problem. Is it stable?
- BGK mode unstable against secondary modes if $\ell = k/k_0 = 0.5$ Guo 95, Lin 04
- Non linear simulations Ghizzo 88, Balmforth 89, Manfredi 97 not clear-cut.
- present work: secondary is sub-harmonic at high dissipation.

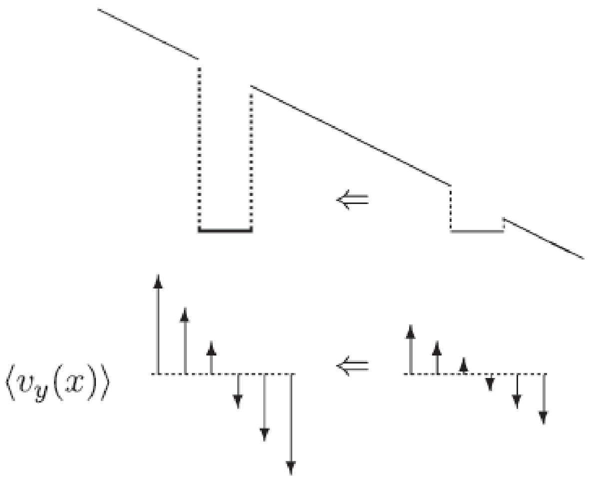


Balmforth 89

An other aside: granulation in phase space turbulence

- Granulation in phase space Dupree 72, 82 , Diamond book festival 11
- Drift holes trigger anomalous resistivity, subcritical instability Lesur 12,13
- Analogy with 2D fluid turbulence → turbulent mixing, zonal flows. Kosuga 11,12.
- Project this year : "Impact of collisions on phase-space structures in tokamak drift-wave turbulence" Artur Kryzhanovskyy



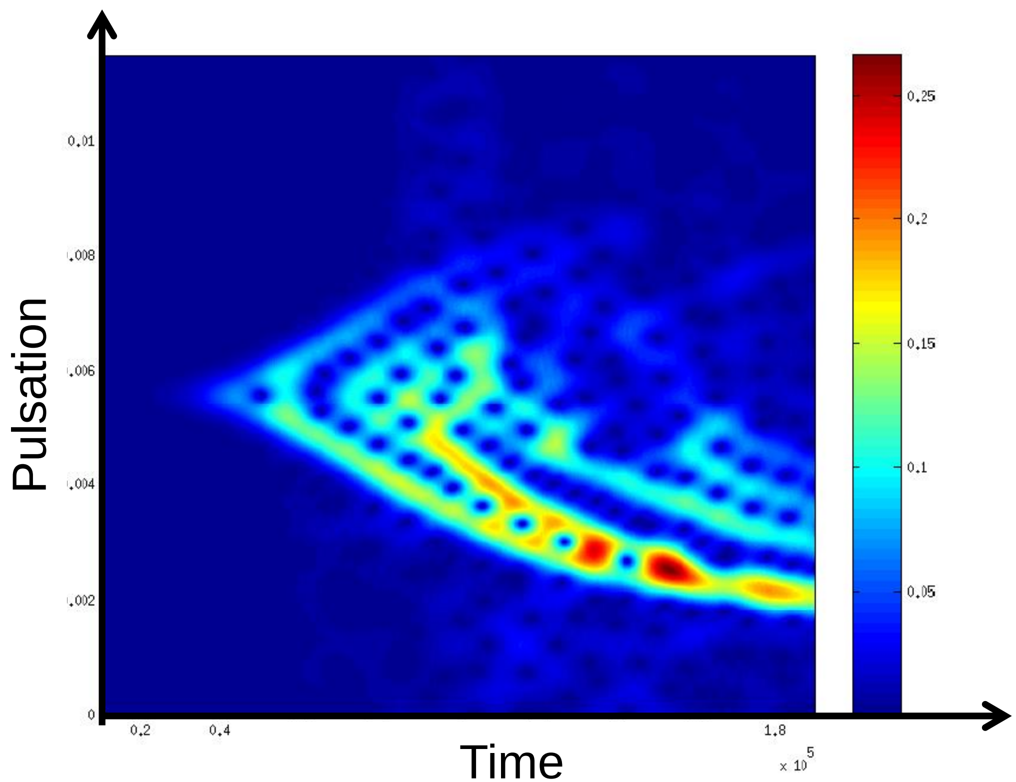


Kosuga 12

An example in tokamaks: fishbones

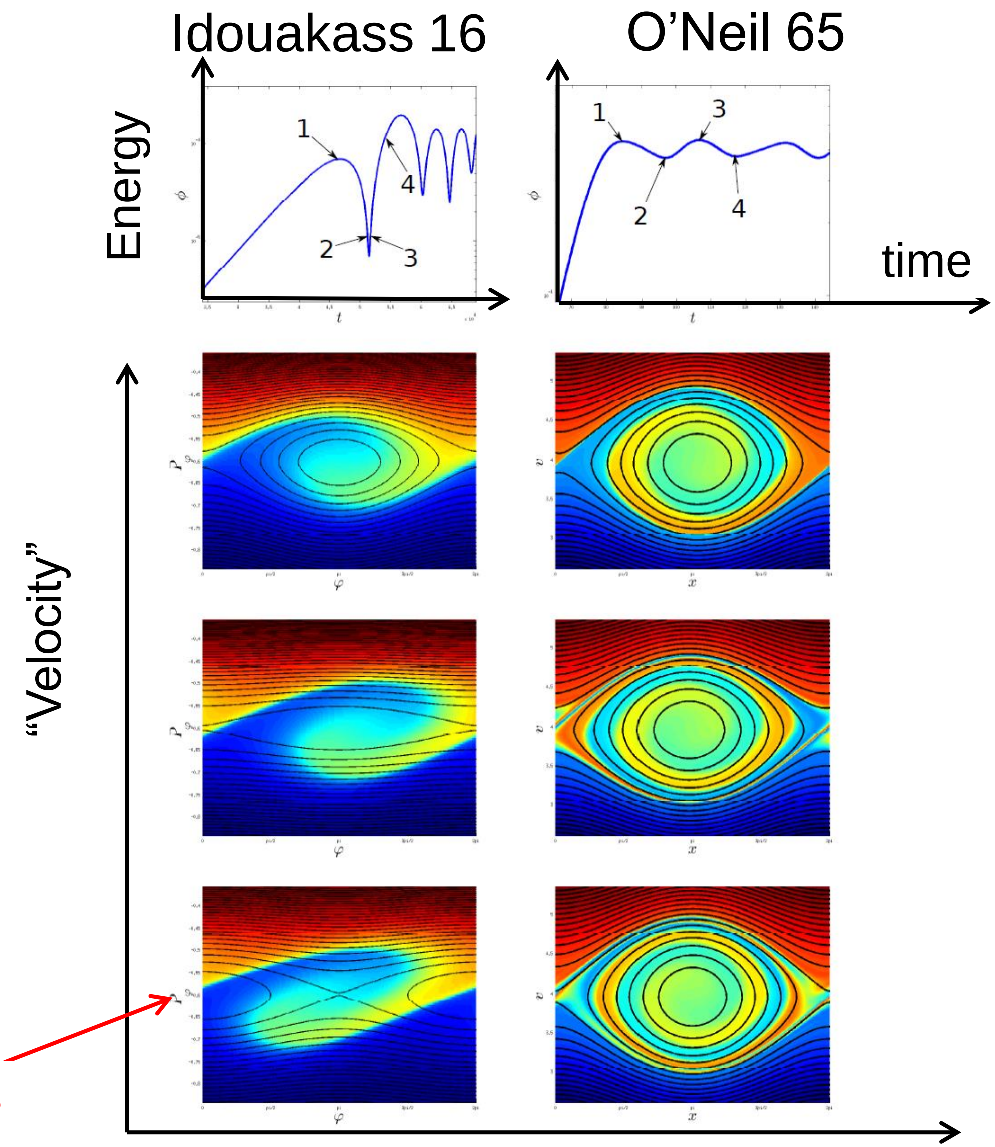
- Fishbone instability: MHD mode driven by fast ions in tokamaks Coppi 86, White 88, Porcelli 91, Fu 06, Pei 17.
- Hybrid reduced model 2D ideal MHD coupled to 2D Vlasov equation.
- Down chirping observed numerically.

Idouakass/Faganello 16



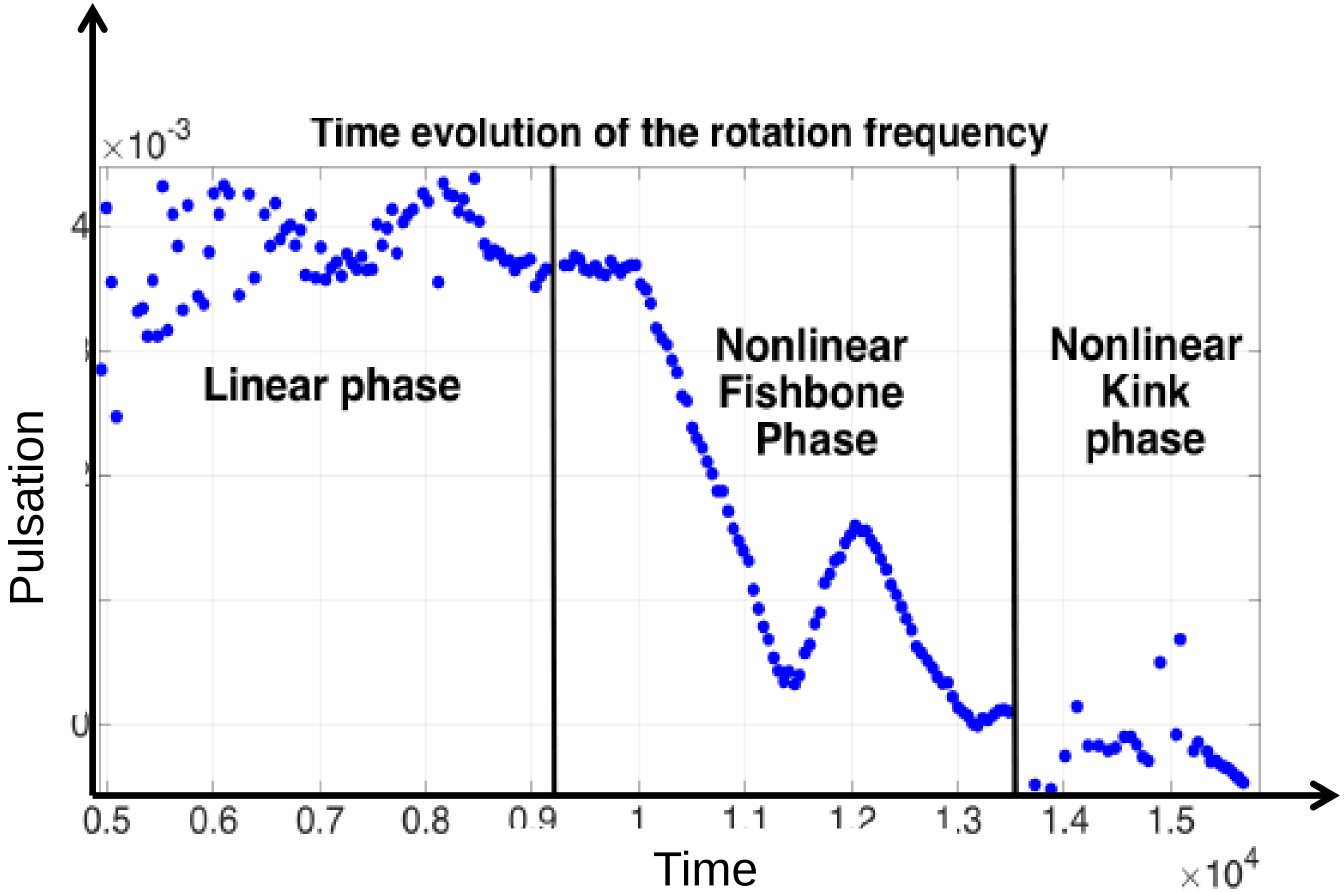
Phase slippage is responsible for chirping in reduced MHD

- Phase slippage: fast shift of the island phase by π
- Roughly agrees with finite time perturbative solution Fokker-Planck/Poisson Berk-Breizman 96



Down chirping is also seen in full MHD.

Full “hybrid” model 3D MHD (thermal electrons + ions) + 6D PIC (fast ions): fishbone down-chirps as well.



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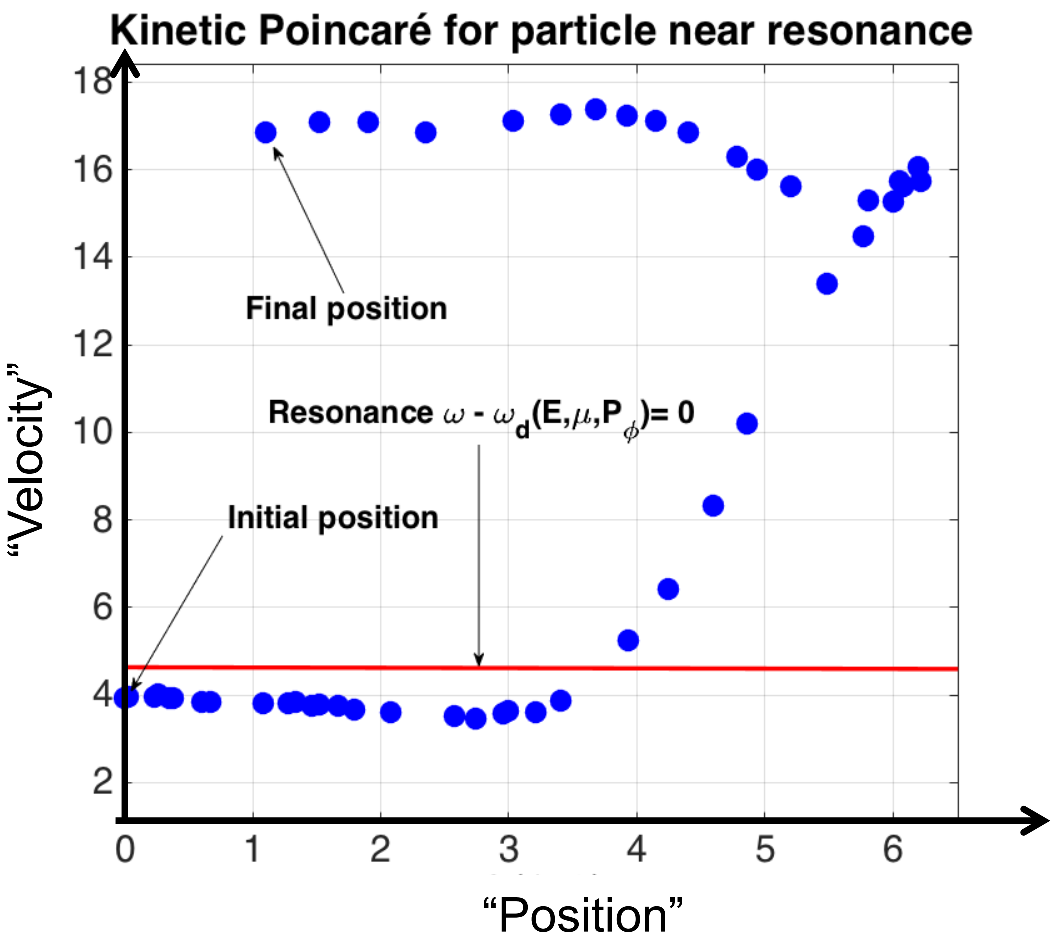
Dynamics can be so fast that trapping does not occur – strong drive.

- Trapping time > fishbone growth time
- Fast dynamics → strong drive? Chen and Zonca 18



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Conclusions

- An island is subject to secondary instabilities.
- However the onset conditions are stringent: secondary modes stable when dissipation is large.
- May explain why chirping is not observed at high collisionality.
- Reduced and full MHD hybrid simulations suggest that model improvement is needed.