Gyro-kinetic theory of turbulent acceleration and momentum conservation

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Outline

- **Part I: (toroidal) momentum transport and intrinsic rotation**

  - **Two review papers:**
    - ✓ **Diamond et at., Nucl. Fusion 53 (2013) 104019**
    - ✓ **Ida and Rice, Nucl. Fusion 54 (2014) 045001**

- **Part II: turbulent acceleration and momentum conservation**

Outline

- **Part I: (toroidal) momentum transport and intrinsic rotation**

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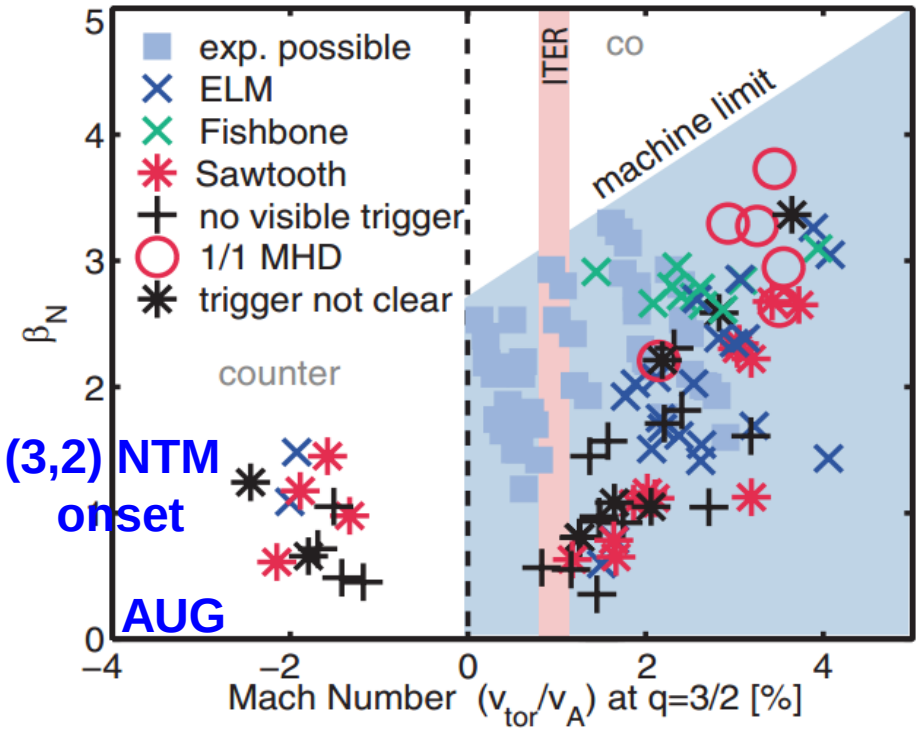
- **Part II: turbulent acceleration and momentum conservation**

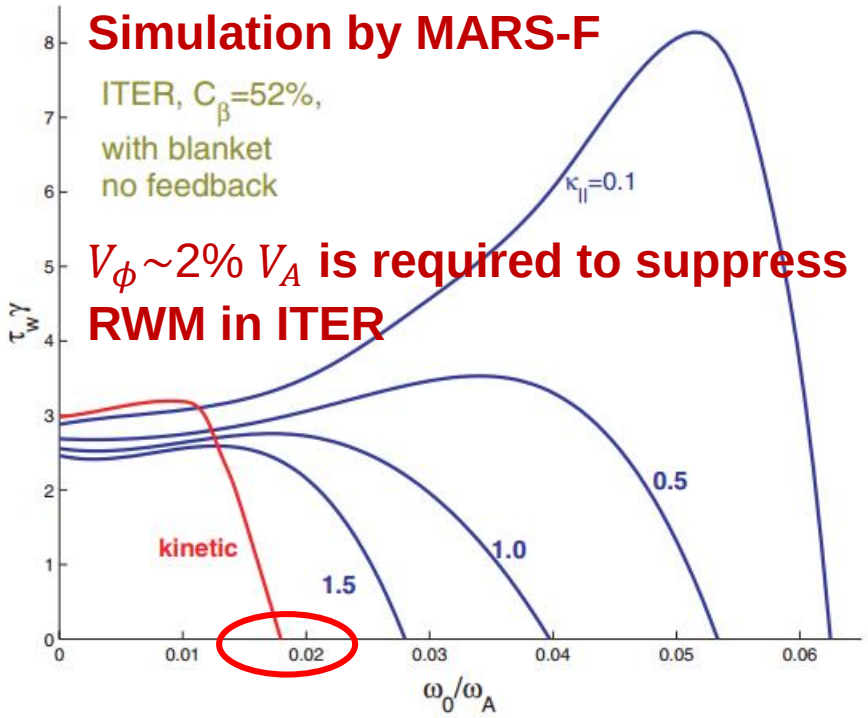
Introduction

- **Understanding of momentum transport is behind of that of particle transport and heat transport**
- **More interest in momentum transport, because toroidal rotation is beneficial to MHD stability (NTM & RWM)**

[Fietz PPCF 55 (2013) 085010]



[Bonderson, PPCF **45** (2003) A253]



Introduction

- **Toroidal rotation also plays a key role in dynamics of confinement improvement**
- **Equilibrium radial force balance**

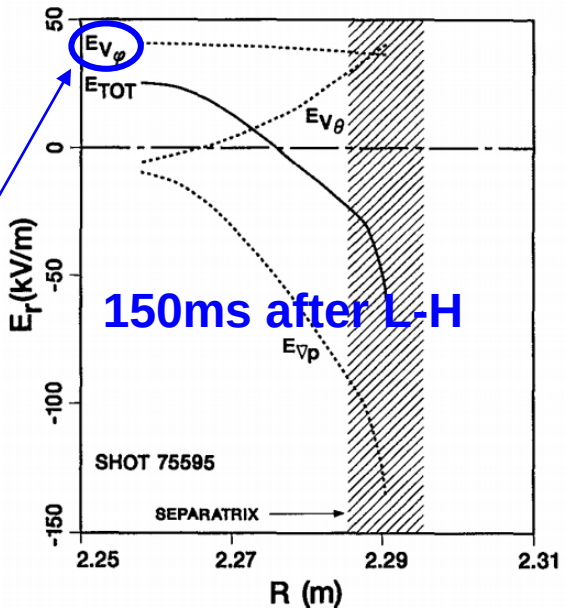
$$E_r = U_\phi B_\theta - U_\theta B_\phi + \frac{1}{e_i Z_i n_i} \nabla_r P_i$$

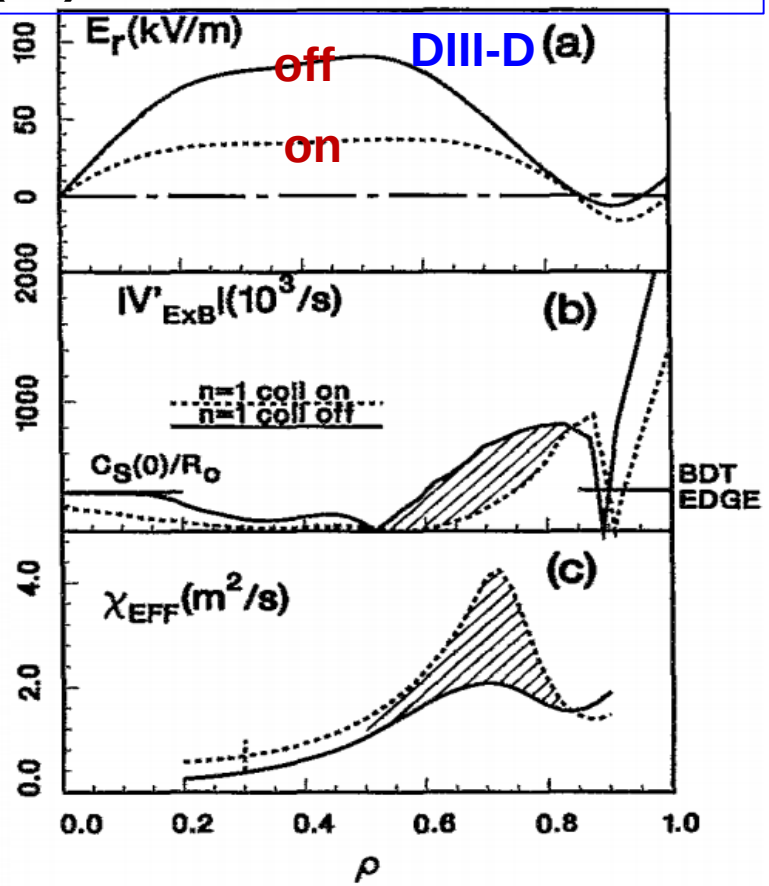
toroidal flow poloidal flow

- **L-H transition: $\nabla_r P_i$ and U_θ dominate @edge**
- **H-VH (very high):**

  U_ϕ dominates @core

U_ϕ higher (lower) by turning off (on) RMP coils





Burrell PoP 1, 1536 (1994)

Introduction

- **Momentum transport equation**

$$\frac{\partial}{\partial t}\langle nU_\phi \rangle + \nabla \cdot \Gamma_{r,\phi} = \tau_\phi$$

- $\Gamma_{r,\phi}$: **turbulent momentum flux**
- τ_ϕ: **torque including driving and damping, external and self-generated**
  - **Driving: neutral beam injection, JxB force (orbit loss, electrode bias), turbulent source/sink**
  - **Damping: NTV (neoclassical toroidal viscosity), charge exchange between bulk ions and neutrals**

Introduction

- **Turbulent momentum flux:**

$$\Gamma_{r,\phi} = \langle n \rangle \langle \tilde{v}_r \tilde{U}_\phi \rangle + \langle U_\phi \rangle \langle \tilde{v}_r \tilde{n} \rangle + \langle \tilde{v}_r \tilde{n} \tilde{U}_\phi \rangle$$

Triplet term usually ignored

- **Reynolds stress divided to terms** $\propto \frac{\partial \langle U_\phi \rangle}{\partial r}$ and $\langle U_\phi \rangle$, **and independent of** $\langle U_\phi \rangle$

$$\Pi_{r,\phi} = \langle \tilde{v}_r \tilde{U}_\phi \rangle = -\chi_\phi \frac{\partial \langle U_\phi \rangle}{\partial r} + V_p \langle U_\phi \rangle + \Pi_{\phi,res}$$

  - χ_ϕ: **Diffusivity (Viscosity)**
  - V_p: **Pinch velocity**
  - $\Pi_{\phi,res}$: **Residual stress – non-diffusive & non-convective term**

Introduction

$$\frac{d}{dt}\frac{1}{r}\int_0^r \langle P_\phi \rangle dr' = nm\left(\chi_\phi \frac{\partial \langle U_\phi \rangle}{\partial r} - V_p(r)\langle U_\phi(r)\rangle - \Pi_{\phi,res}(r)\right) + \frac{1}{r}\int_0^r r'\tau_\phi dr'$$

Diffusive flux
Co: $\langle U_\phi \rangle$'<0→ctr torque
Ctr: $\langle U_\phi \rangle$'>0→co torque

Flatten

Pinch (convective) flux
Can be in or out

Flatten or steepen

Residual stress & momentum source/sink

Intrinsic rotation drive

- **Rotation profile is a key:**
  - **Diffusion**
  - **Pinch**
- **External momentum drive may not efficient in future reactor:**
  - **Intrinsic rotation**

Outline

- **Part I: (toroidal) momentum transport and intrinsic rotation**
  - Introduction
  - **Reynolds stress**
    - **Diffusion**
    - **Pinch**
  - Intrinsic rotation
    - Residual stress
    - Beyond residual stress
- Part II: turbulent acceleration and momentum conservation

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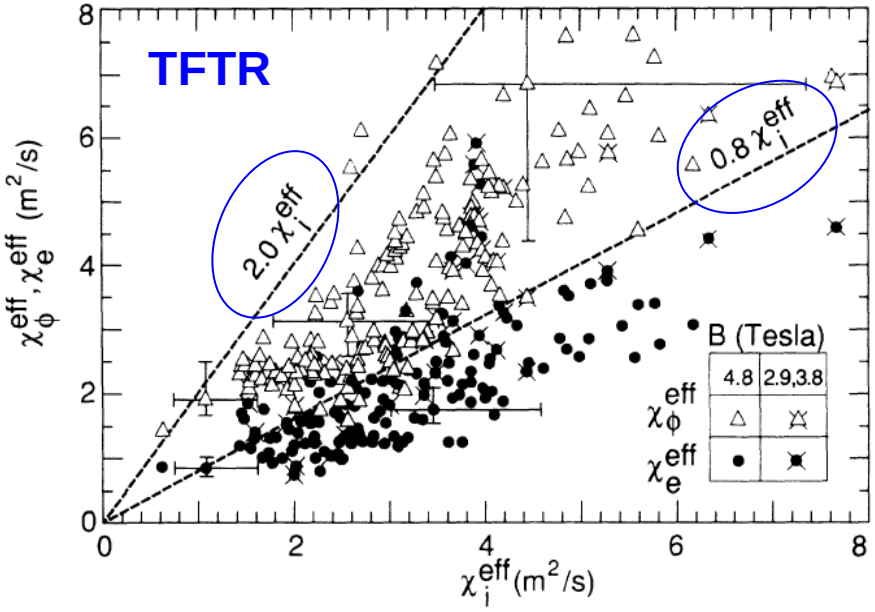
Reynolds stress - diffusion

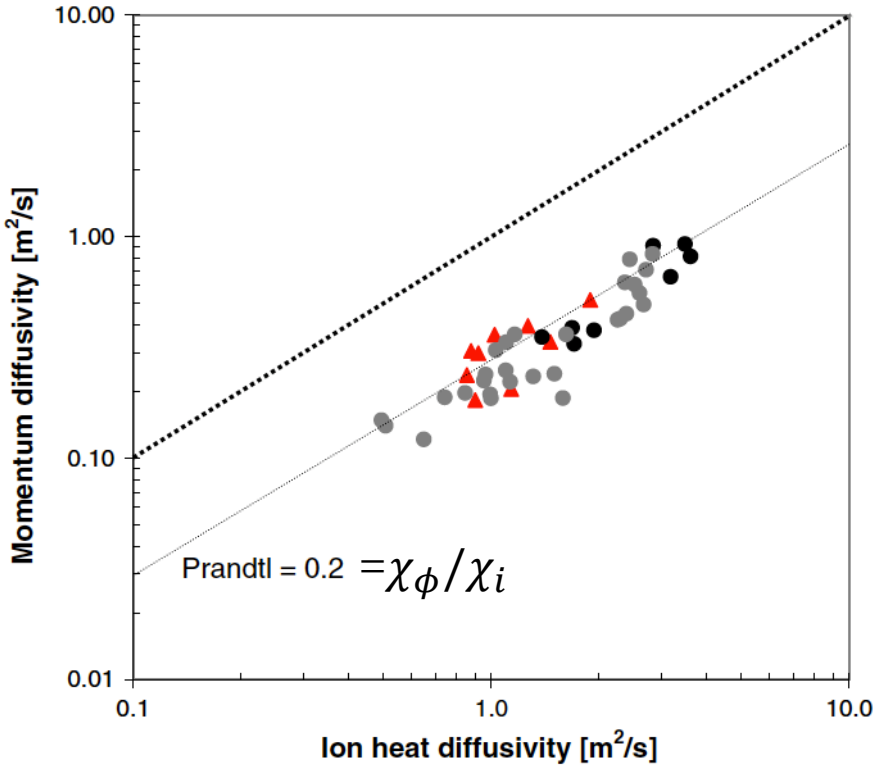
- $\Pi_{r,\phi} = -\chi_\phi \frac{\partial \langle U_\phi \rangle}{\partial r} + V_p \langle U_\phi \rangle + \Pi_{\phi,res}$
- **Early 90's: toroidal momentum diffusivity (viscosity) is comparable to heat diffusivity, $\chi_\phi / \chi_i \sim 0.8 - 2$**
- **Rough agreement with theoretical predilections based on ES ITG or TEM turbulence**



[Scott S.D. et al, 1990 Phys. Rev. Lett. **64** 531]

Reynolds stress - diffusion

➢ **Different observations on JET** [de Vries, PPCF 2006 **48** 1693]



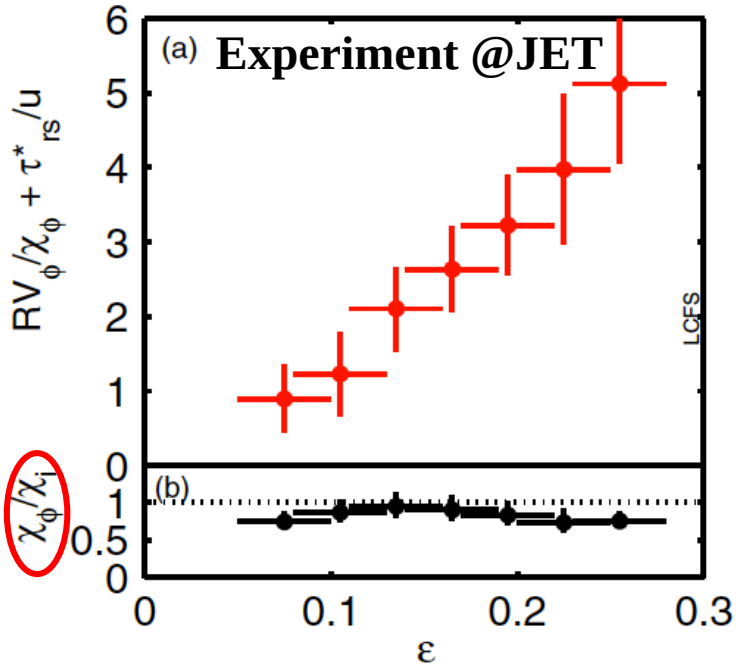
- χ_ϕ/χ_i**~0.2**
- **But** $\frac{\tau_\phi}{\tau_E}\sim 1$
- **Why? → Non-diffusion →**

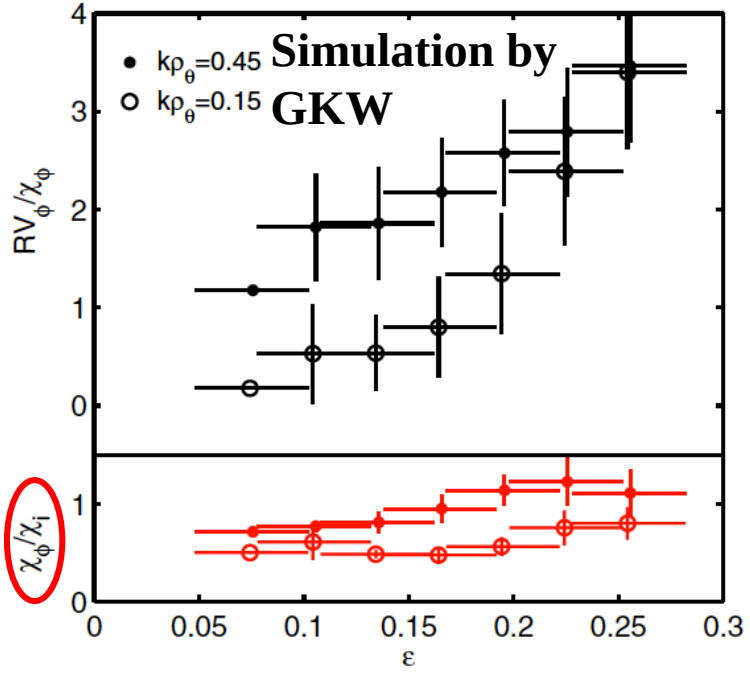
  Momentum pinch? &

  Possible heat pinch?

Reynolds stress - pinch (convection)

- $\Pi_{r,\phi} = -\chi_\phi \frac{\partial \langle U_\phi \rangle}{\partial r} + V_p \langle U_\phi \rangle + \Pi_{\phi,res}$ [de Vries, PPCF 2006 **48** 1693]



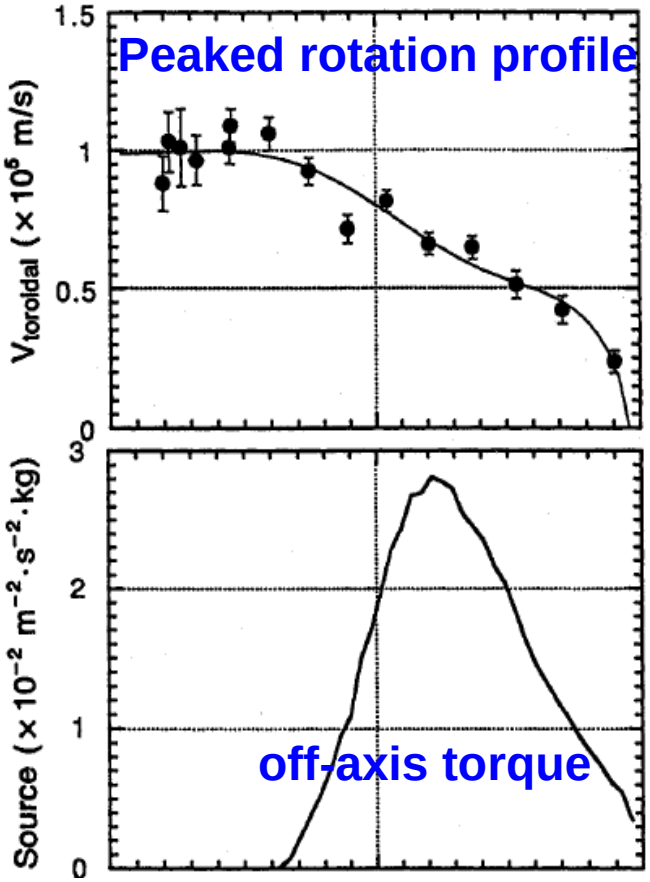


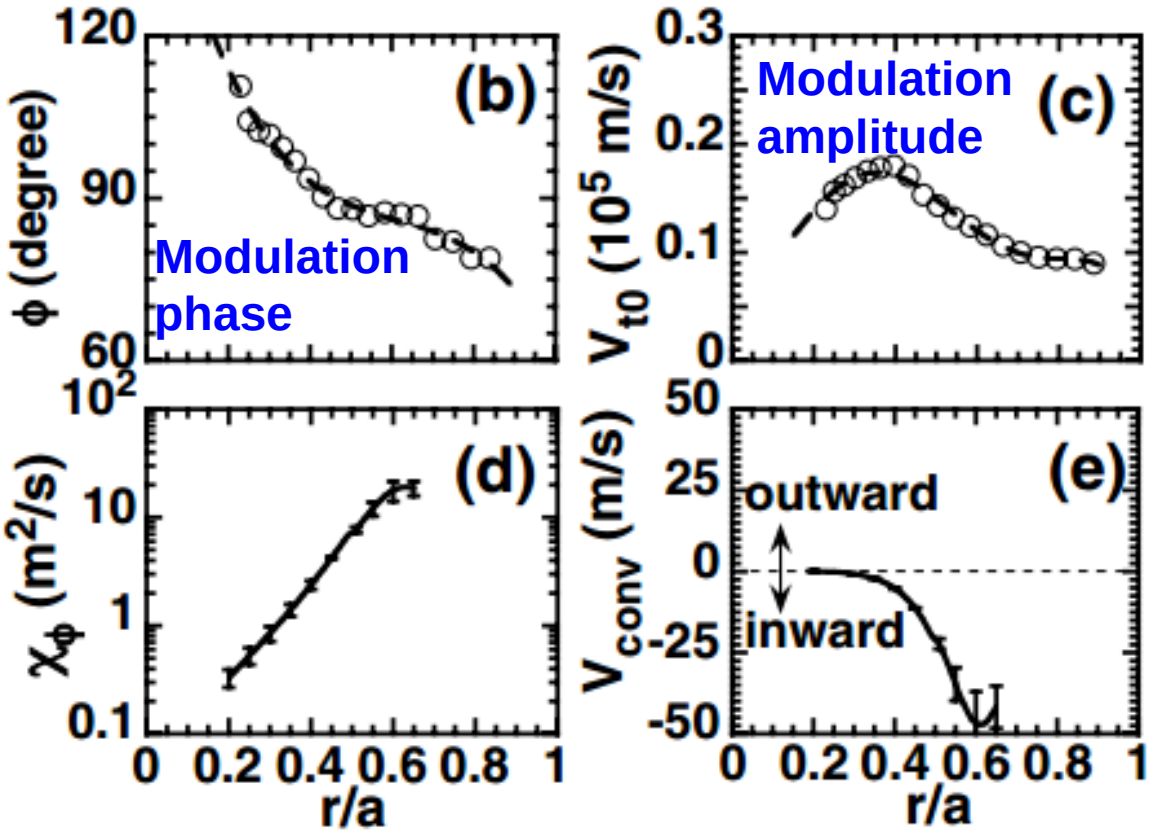
- $\frac{\chi_\phi}{\chi_i} \sim 1$ **after including momentum pinch**
- **Agreement between experiments and simulation**

Reynolds stress - pinch (convection)

- $\Pi_{r,\phi} = -\chi_\phi \frac{\partial \langle U_\phi \rangle}{\partial r} + V_p \langle U_\phi \rangle + \Pi_{\phi,res}$

JT-60U





[Yoshida M. et al 2008 PRL 100 105002]

[K.Nagashima et al., NF 34 (1994) 449]

Momentum pinch - theory

- **TEP (Turbulent Equipartition) pinch from gyrokinetic theory**
  (Hahm PoP 2007, 2008, Gurcan et al., PRL 2008)

$$(\partial_t + \mathbf{v}_E \cdot \nabla)\left(\frac{nU_\parallel R}{B^2}\right) \approx 0$$

magnetically weighted angular momentum density is a locally advected scalar due to toroidal geometry

→ angular momentum pinch — similar to TEP particle pinch

$$V_{\mathrm{Ang}}^{\mathrm{TEP}} \simeq -\frac{2F_{\mathrm{balloon}}}{R_0}\chi_{\mathrm{Ang}}$$

- **Coriolis pinch from fluid theory [Peeters PRL 98, 265003 (2007)]**

$$\frac{RV_\phi}{\chi_\phi} = -4 - R/L_n$$

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- **Part I: (toroidal) momentum transport and intrinsic rotation**
  - Introduction
  - Reynolds stress
    - Diffusion
    - Pinch
  - **Intrinsic rotation**
    - **Residual stress**
    - **Beyond residual stress**
- **Part II: turbulent acceleration and momentum conservation**

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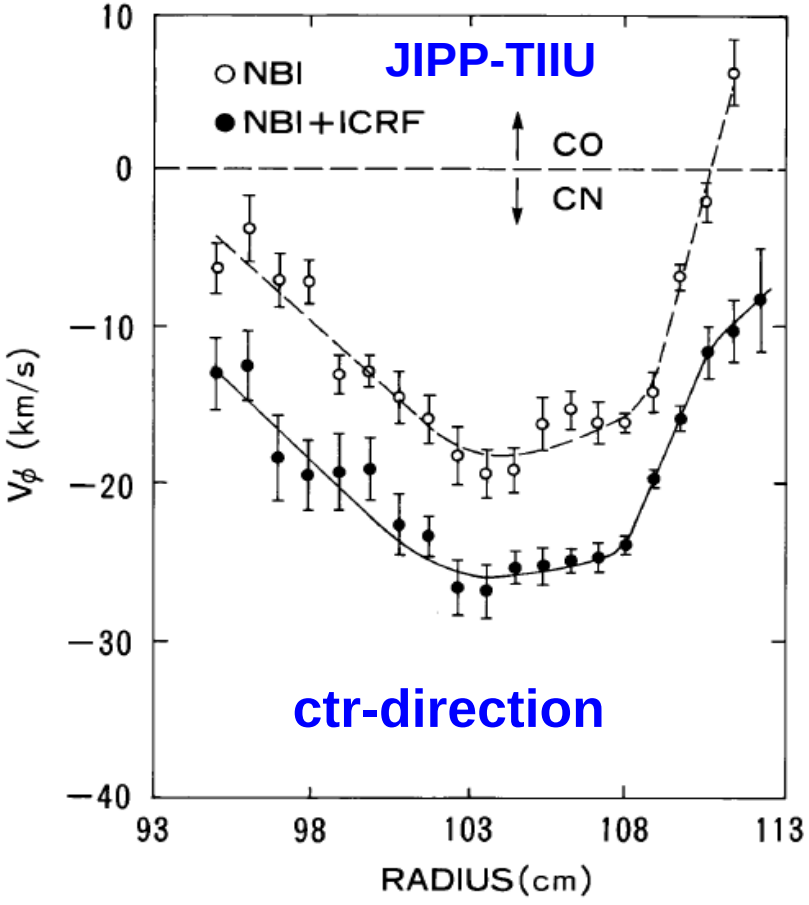
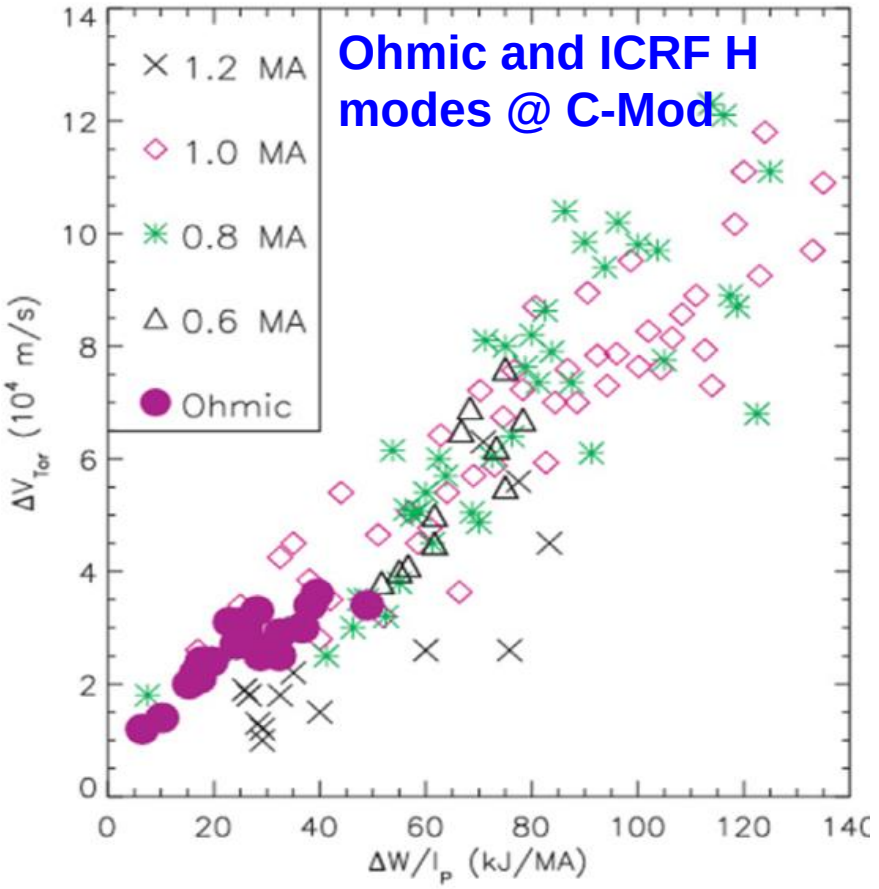
Intrinsic rotation

- (change of) rotation without direct external torque



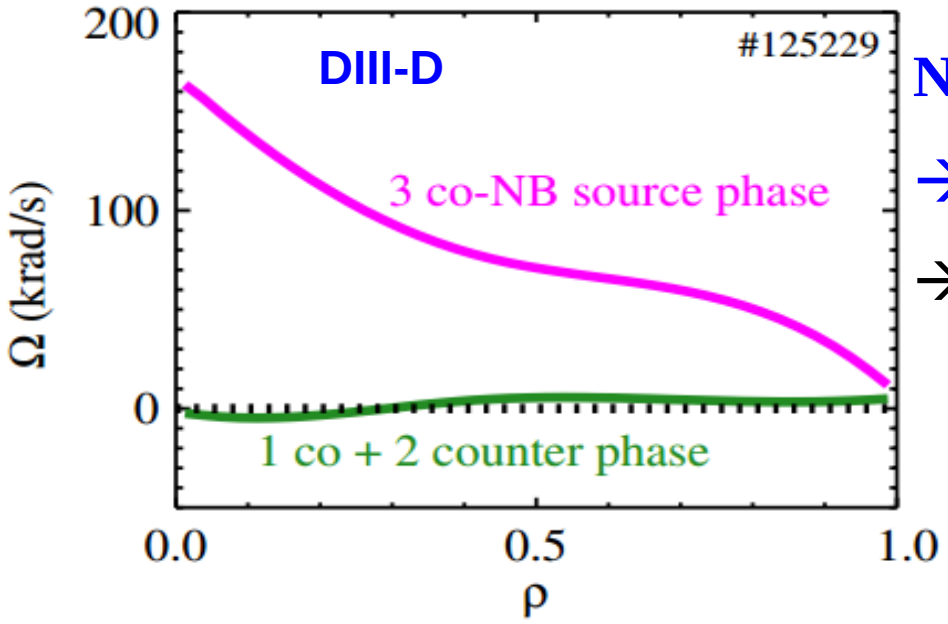
[Ida, Nucl. Fusion 31 (1991) 943]



[Rice, Nucl. Fusion 38 (1998) 75]

Intrinsic rotation

- **Zero toroidal rotation with unbalanced NBI torque**



No rotation with external torque

→ Intrinsic torque ~ 1 co NBI torque

→ Momentum pinch cannot explain due to zero rotation

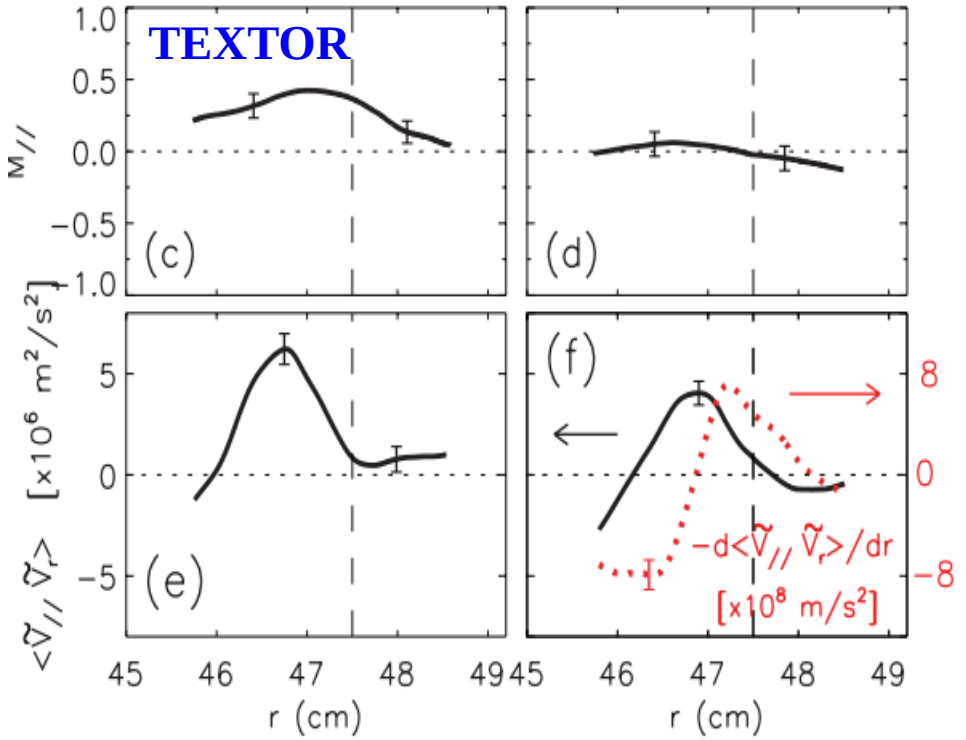
[Solomon, 2007 PPCF 49 B313]

net momentum input to the plasma from the imbalance of neutral beams. Hence, we must either have an anomalous source of torque in the plasma balancing out the residual from the 2 counter+1 co neutral beam sources, or alternatively we are not accounting for all possible momentum fluxes (e.g. other off diagonal terms, or the residual stress [21]). As

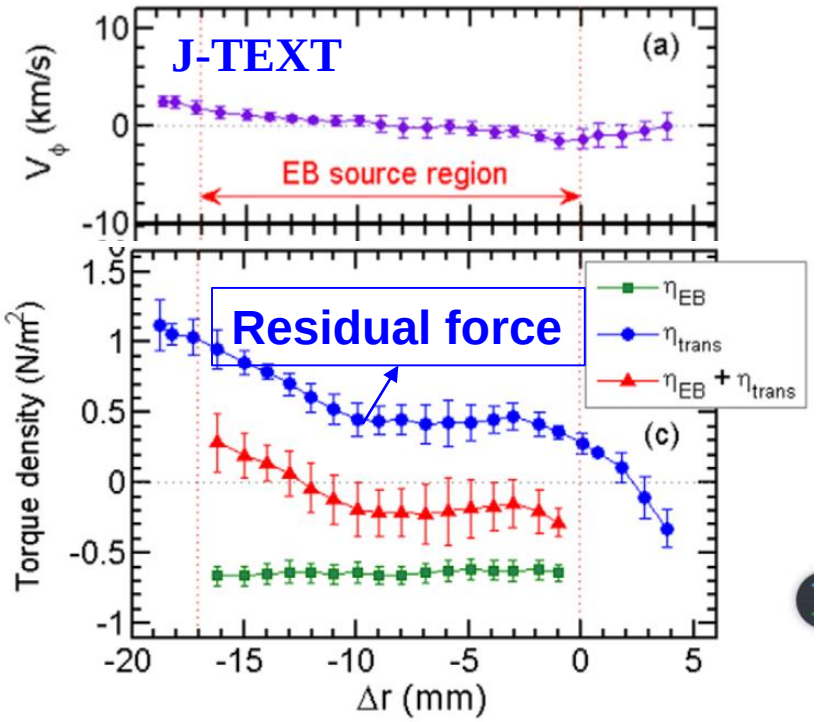
Intrinsic rotation – residual stress

- $\Pi_{r,\phi} = -\chi_\phi \frac{\partial \langle U_\phi \rangle}{\partial r} + V_p \langle U_\phi \rangle + \Pi_{\phi,res}$
- **Experimental evidence of residual stress**

Without NBI **with ctr-NBI**



[Xu, NF **53** (2013) 072001]

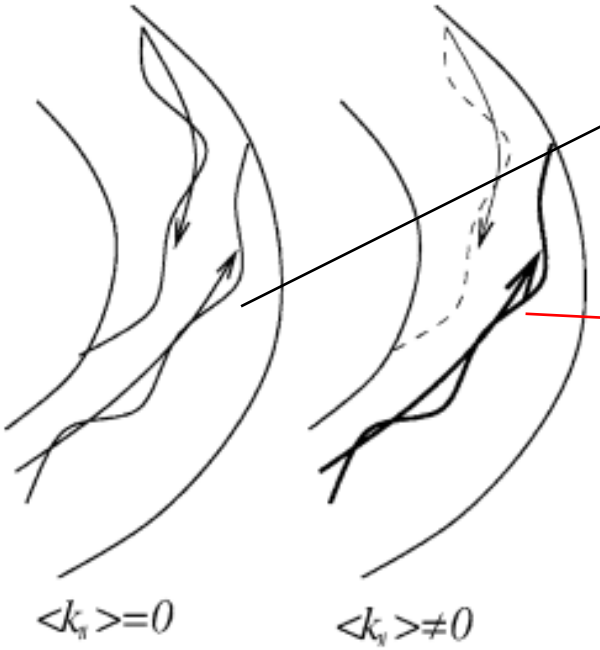


[Sun, NF 56 (2016) 046006]

Intrinsic rotation – residual stress

- $\Pi_{r,\phi} = -\chi_\phi \frac{\partial \langle U_\phi \rangle}{\partial r} + V_p \langle U_\phi \rangle + \Pi_{\phi,res}$
- $\Pi_{\phi,res} = \langle \tilde{v}_r \tilde{v}_\parallel \rangle^{\mathrm{R}} \propto \sum_k k_y k_\parallel |\phi_k|^2 \propto \sum_k k_y^2 \frac{x}{L_s} |\phi_k|^2$

→ $k_\parallel$ **symmetry breaking is required for non-zero residual stress**



Symmetric: balanced counter-propagating wave populations → $\Pi_{\phi,res} \sim 0$

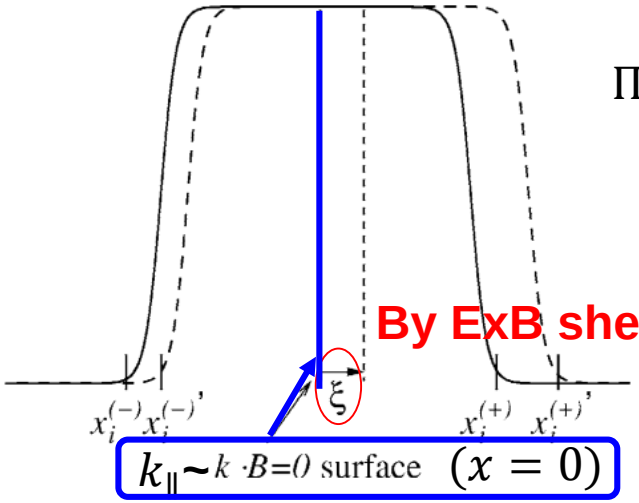
Asymmetric: imbalanced counter balanced counter-propagating wave populations → $\Pi_{\phi,res} \neq 0$

What are the mechanisms?
-- asymmetric turbulence intensity

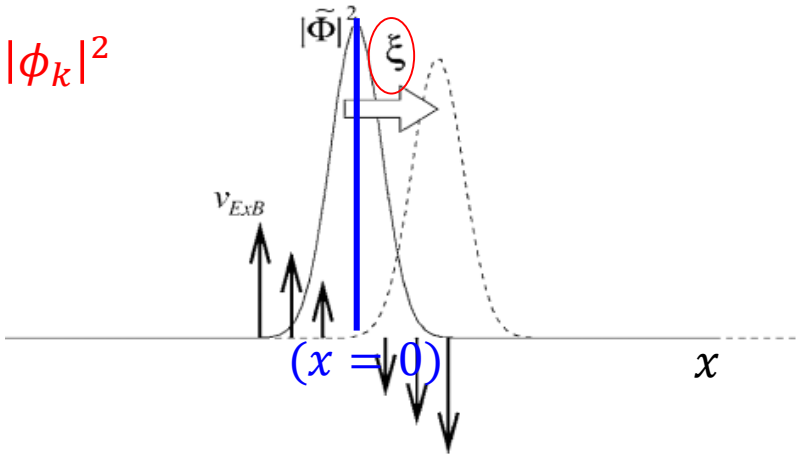
[Gurcan, PoP 14, 042306 (2007)]

Intrinsic rotation – residual stress

- $k_\parallel$ symmetry breaking by ExB shear **Gurcan, PoP 14, 042306 (2007)**



$$\Pi_{\phi,res} \propto \sum_k k_y^2 \frac{x}{L_s} |\phi_k|^2$$



Shifts in the ion Landau resonant points

Shifts of the turbulence intensity

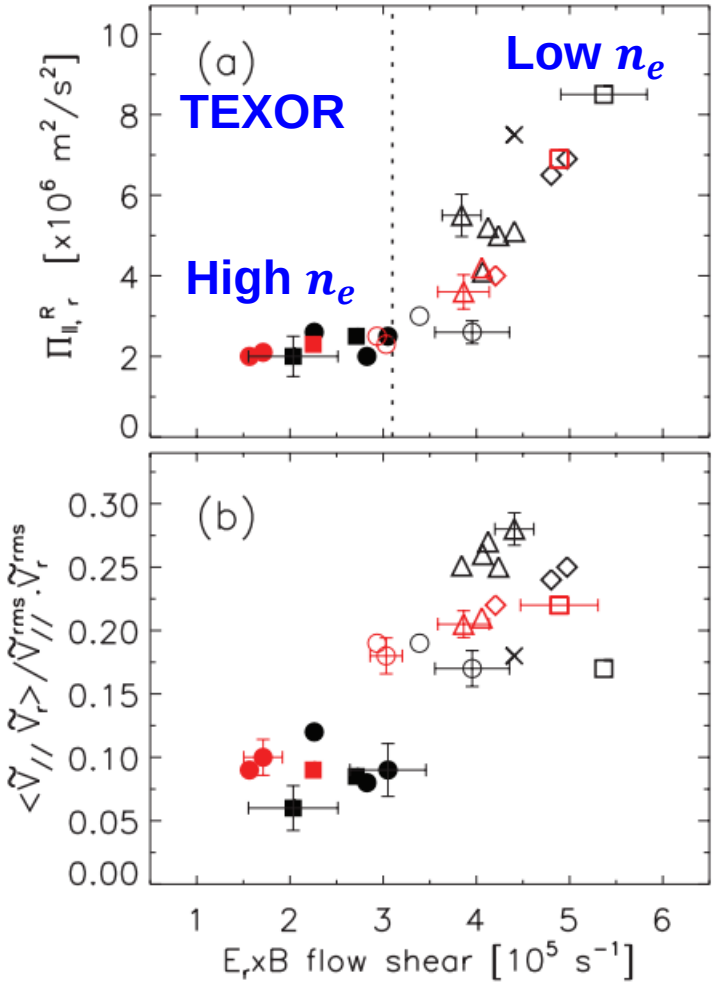
- **Without ExB shear:** $\omega \mp k_\parallel v_i = 0 \xrightarrow{k_\parallel = k_y \frac{x}{L_s}} x_i^{\pm}/L_s = \pm \frac{\omega}{k_y v_i}$ →**symmetry**
- **With shear:** $\omega - k_y V'_{E\times B} x \mp k_\parallel v_i = 0 \rightarrow \frac{x_i^{\pm\prime}}{L_s} = \pm \frac{\omega}{k_y v_i (1 \pm V'_{E\times B} \frac{L_s}{v_i})}$ →**asymmetry**

→**shift**: $\frac{\xi}{L_s} = \frac{(x_i^{+\prime} + x_i^{-\prime})/2}{L_s} = -\frac{\omega_r}{\omega_{*e}} V'_{E\times B} \frac{L_s}{L_n} \frac{\rho_i}{v_i} \tau$

Sign depends on the turbulence modes

Intrinsic rotation – residual stress

➢ $k_{\|}$ symmetry breaking by ExB shear – experimental evidence

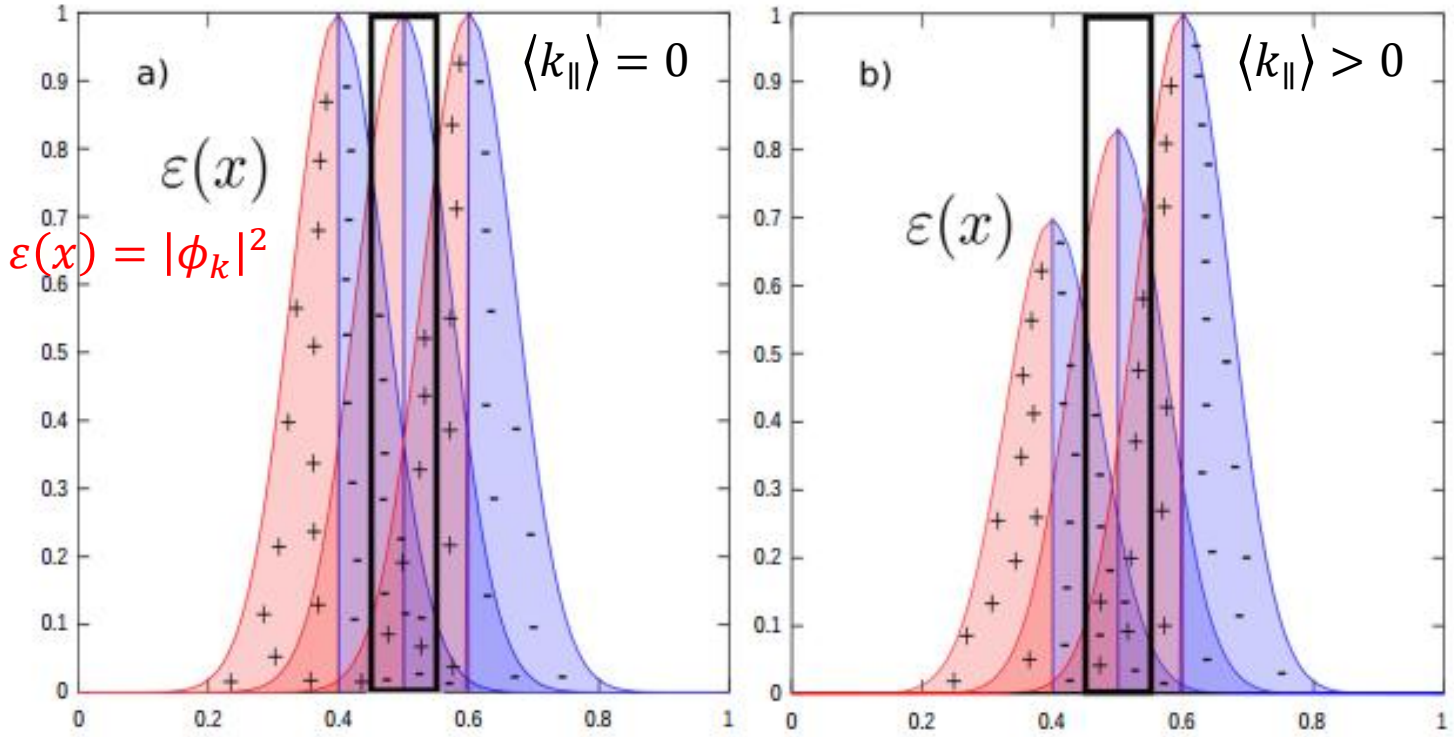


- **Maximum residual stress vs the local $E\times B$ shear rate →**
  - **Critical shear rate for $k_{\|}$ symmetry breaking**
  - **Residual stress increases with increasing shear rate**
- **Higher $E\times B$ shear rate → higher cross correlation**

[Xu, NF **53** (2013) 072001]

Intrinsic rotation – residual stress

- $k_\parallel$ **symmetry breaking by intensity gradient**



- $\Pi_{\phi,res} = \langle \tilde{v}_r \tilde{v}_\parallel \rangle^{\mathrm{R}} \propto \sum_k k_y k_\parallel |\phi_k|^2 \propto \sum_k k_y^2 \frac{x}{L_s} \varepsilon(x)$

$$\varepsilon(x) = \varepsilon(0) + x \left. \frac{d\varepsilon(x)}{dx} \right|_{x=0}$$

$\Rightarrow \Pi_{\phi,res} \neq 0$

[Gurcan, PoP 17, 112309 (2010)]

Intrinsic rotation – residual stress

Selective Symmetry Breaking & Physics of symmetry breaking mechanisms

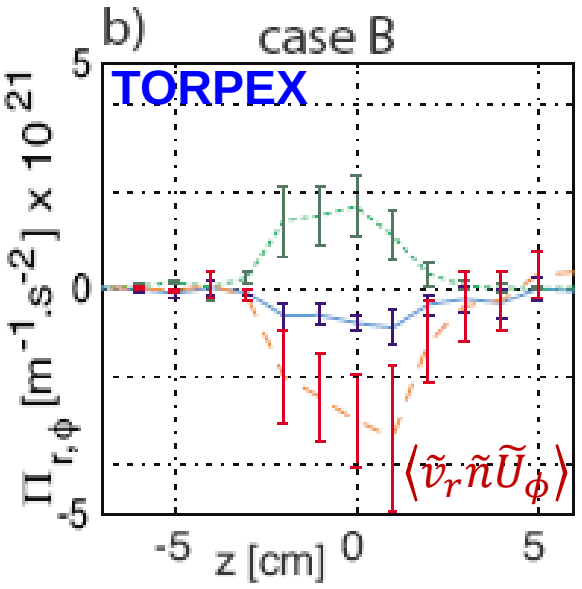
[Diamond, NF 53 (2013) 104019]

Relevant Stress and Mechanism	Spatial Structure	Key Physics	Macro Implication
$\langle \tilde{v}_r \tilde{v}_\parallel \rangle$, $\langle v_E \rangle'$ (Electric field shear)	$k_\parallel$ from spectrum *shift* (config.) or *eddy tilt* (ballooning)	Centroid shift induces mean $\langle k_\parallel \rangle$ from 1 direction acoustic wave asymmetry	$\pi_{res} \sim \langle v_E \rangle'$. Intrinsic torque peaked at barriers, steep gradients π_{res} can flip with mode change
$\langle \tilde{v}_r \tilde{v}_\parallel \rangle$, I' (Intensity gradient)	$k_\parallel$ from spectra *dispersion* due I'	Spectral dispersion from intensity gradient. Linked to $\perp$ Reyn. stress	$\pi_{res} \sim I'$. relevant to barriers but also for more general inhomogeneity. Can change with mode change. Related to temp. profile curvature
Stress from polarization acceleration $\langle \tilde{E}_\parallel \nabla_\perp^2 \tilde{\phi} \rangle$	$\langle k_r k_\parallel \lvert \phi_{\mathbf{k}} \rvert^2 \rangle$ stress due radial + parallel propagation, $(r, \parallel)$ tilting	G.C. stress from acceleration due polarization charge $\langle k_r k_\parallel \rangle \neq 0$ needed	As yet unclear. Merits further study. Linked to mode $v_g r$ and can flip
Stress from $\partial_r \langle \tilde{v}_r \tilde{v}_\perp \rangle$ $\rightarrow \langle J_r \rangle \rightarrow B_\theta \langle J_r \rangle$ $\rightarrow$ toroidal torque	(r, θ) tilting, as for ZF Same as for ZF	$\mathbf{J} \times \mathbf{B}$ torque originating from polarization flux $I' \neq 0$, $\langle k_r k_\theta \rangle \neq 0$ needed	$\sim$ universal mechanisms, closely related to ZF, tied to I' and $I_{\mathbf{k}}$ structure. Flips with v_{gr}. Merits more study.

Intrinsic rotation – beyond residual stress

➢ **Total momentum flux:** $\Gamma_{r,\phi} = \langle n \rangle \langle \tilde{v}_r \tilde{U}_\phi \rangle + \langle U_\phi \rangle \langle \tilde{v}_r \tilde{n} \rangle + \langle \tilde{v}_r \tilde{n} \tilde{U}_\phi \rangle$

- **Exp: nonlinear toroidal momentum flux is strongly increased.**

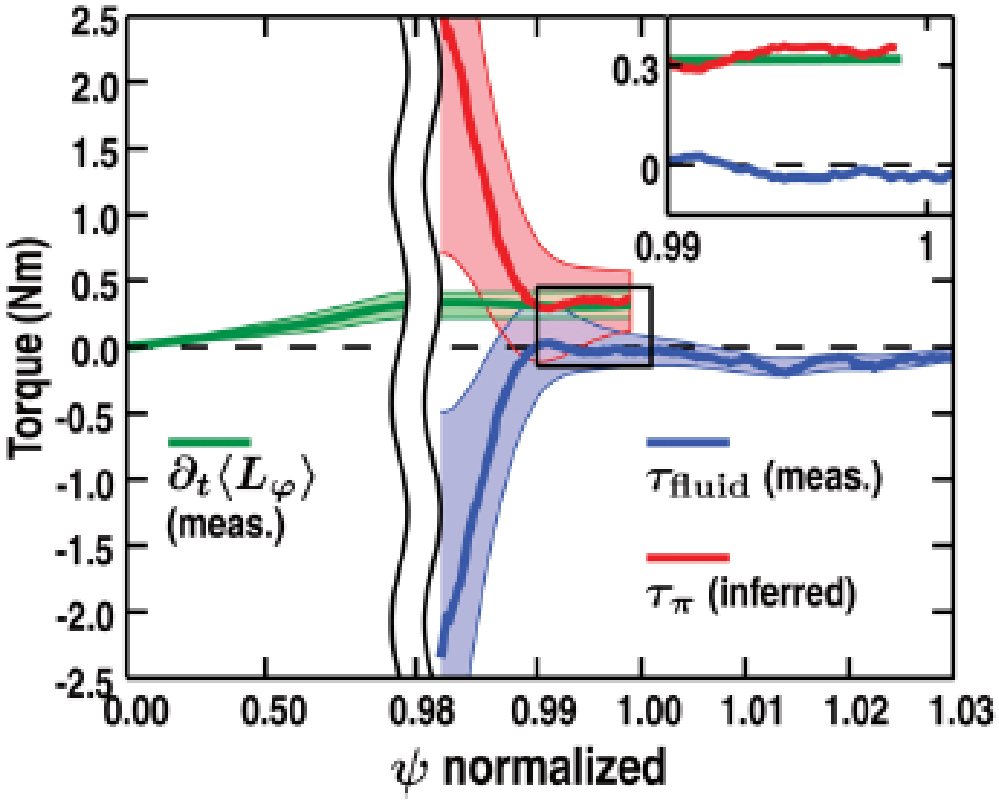


[Labit et al., PoP 2011]

- **Theory: May contribute to intrinsic torque in strong turbulence regime, such as blob ejection at tokamak edge plasmas**
  - ✓ **Wang, Wen and Diamond, PoP 2015**
  - ✓ **Kosuga, Itoh and Diamond, PRE 2017**
  - ✓ **Li, Li and Gao, NF 2019**

Intrinsic rotation – beyond residual stress

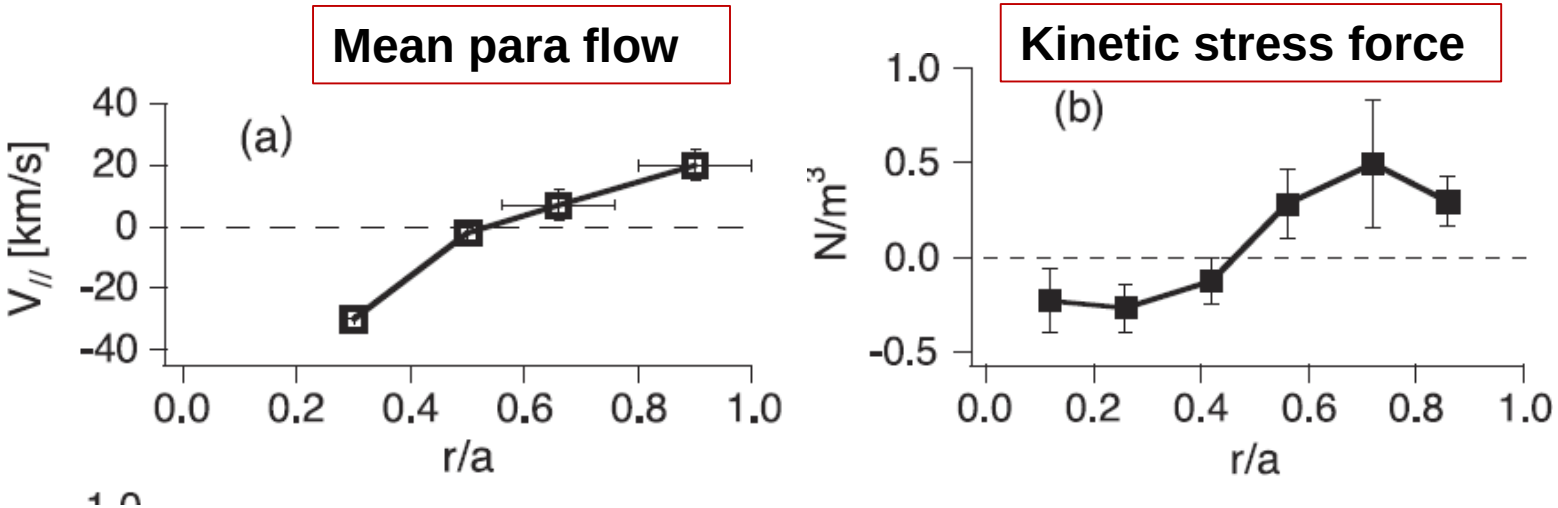
- **DIII-D: The torque from fluid Reynolds stress cannot explain the edge rotation profile [Muller et al., PoP 18, 072504 (2011)]**



Kinetic stress may be needed to explain the discrepancy

Intrinsic rotation – beyond residual stress

- **MST: kinetic stress driven intrinsic parallel flow has been observed** [W.X. Ding et al., PRL 110, 065008 (2013)]



What's kinetic stress?

Contribute to intrinsic torque?

→ Part II

Intrinsic rotation – beyond residual stress

➢ Turbulent acceleration – self-generated torque (force)

❑ Turbulent source for angular momentum

(Garbet, PoP 20, 072502 (2013))

$$\partial_t \mathcal{L}_\varphi + \partial_\chi \Pi^\chi_\varphi = R_0 M_\parallel,$$

❑ Turbulent source for parallel flow velocity

(Wang & Diamond, PRL 110, 265006 (2013))

$$\frac{\partial}{\partial t}\langle U_\parallel \rangle + \nabla \cdot \Pi_{r,\parallel} = a_\parallel$$

→ Part II

Outline

Part II: turbulent acceleration and momentum conservation

- **Turbulent acceleration of parallel flow**
  - **Intrinsic flow drive by E.S. and E.M. ITG turbulence**
- **Consistency between turbulent acceleration and momentum conservation**
- **Summary**

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  - **Intrinsic flow drive by E.S. and E.M. ITG turbulence**
- **Consistency between turbulent acceleration and momentum conservation**
- **Summary**

Gyrokinetic approach

- **Starting from E.S. gyrokinetic equations** [Hahm, Phys. Fluids 1988]

$$\frac{\partial}{\partial t}\left(FB^*\right)+\nabla\cdot\left(\frac{d\mathbf{R}}{dt}FB^*\right)+\frac{\partial}{\partial v_\parallel}\left(\frac{dv_\parallel}{dt}FB^*\right)=0$$

Gyrocenter distribution function **Gyrocenter EoM**

- **Taking the (0th and 1st order) moments to obtain equations for**
  - **Gyrocneter density by** $\frac{2\pi}{m_i}\int d\mu dv_\parallel$

$$\frac{\partial}{\partial t}n+\nabla\cdot\left[\left(U_\parallel\hat{\mathbf{b}}+\mathbf{v}_{E\times B}+\mathbf{v}_{d\kappa}+\mathbf{v}_{d\nabla}\right)n\right]=0$$

  - **Gyrocenter parallel momentum per ion mass by** $\frac{2\pi}{m_i}\int v_\parallel d\mu dv_\parallel$

$$\frac{\partial}{\partial t}\left(nU_\parallel\right)+\nabla\cdot\left[\frac{P_i}{m_i}\hat{\mathbf{b}}+\left(\mathbf{v}_{E\times B}+3\mathbf{v}_{d\kappa}+\mathbf{v}_{d\nabla}\right)nU_\parallel\right]$$
$$=\left[-\left[\frac{e}{m_i}\hat{\mathbf{b}}\cdot\nabla\delta\phi+\frac{c}{B}\hat{\mathbf{b}}\times\left(\hat{\mathbf{b}}\cdot\nabla\hat{\mathbf{b}}\right)\cdot\nabla\delta\phi U_\parallel\right]n.\right. \quad (7)$$

Momentum source

Similar to [X. Garbet, PoP 2013]

Turbulent acceleration

➢ **Mean parallel flow equation:** $\frac{\partial}{\partial t}\langle U_\parallel \rangle = \left\langle \frac{1}{n}\left[\frac{\partial}{\partial t}(nU_\parallel) - U_\parallel \frac{\partial}{\partial t} n\right]\right\rangle$

$$\frac{\partial}{\partial t}\langle U_\parallel \rangle + \nabla \cdot \Pi_{r,\parallel} = a_\parallel$$

Decouple density from momentum

$$a_\parallel = \frac{1}{m_i n_0}\left\langle \delta n \hat{\mathbf{b}} \cdot \nabla \delta T_i \right\rangle - 2\left\langle \frac{\delta T_i}{T_i}\mathbf{v}_{d\nabla} \cdot \nabla \frac{\delta n}{n_0}\right\rangle \langle U_\parallel \rangle + \left\langle \delta U_\parallel \mathbf{v}_{d\nabla} \cdot \nabla \left(\frac{e\delta\phi}{T_i} - 2\frac{\delta n}{n_0} - 2\frac{\delta T_i}{T_i}\right)\right\rangle . \quad (10)$$

- **Local source/sink—turbulent acceleration**
- **Ignored in previous works**
- **First term is dominant, and origins from parallel gradient of ion pressure fluctuations**
- **It always exists, whether the magnetic geometry is toroidal or cylindrical**

Turbulent acceleration for ITG turbulence

$$a_{\parallel} \simeq v_{thi}^2 \left\langle \delta\hat{n}\vec{b} \cdot \nabla \delta\hat{T}_i \right\rangle$$

- **By using adiabatic approximation $\delta\hat{n} = \frac{T_{i0}}{T_{e0}}\delta\hat{\phi}$ and linearizing the ion temperature evolution equation $\delta\hat{T}_i = f\left(\nabla T_i, \delta\hat{\phi}\right)$**

- **Parallel symmetry breaking is required. Intensity gradient induced symmetry breaking is taken [Gurcan, PoP 2010]**

$$a_{\parallel} = \frac{T_i}{T_e}\frac{\rho_i}{L_{T_i}} v_{thi}^3 \frac{\hat{s}}{qR} \sum_k (\mathrm{Re}\,\tau_{ck}) k_\theta^2 x^2 \frac{\partial I_k}{\partial x}$$

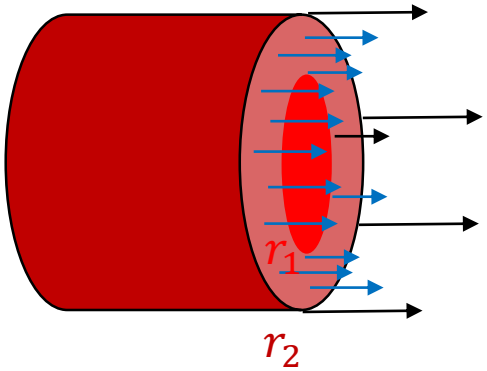
- **If $\hat{s} > 0$ and $\frac{\partial I_k}{\partial x} > 0$, $a_{\parallel} > 0$ co-current drive**

- **A new candidate for the origin of intrinsic flow--independent of flow and flow gradient**

Turbulent acceleration vs residual stress

➢ **Mean flow equation:** $\frac{\partial}{\partial t}\langle U_{\parallel}\rangle + \nabla \cdot \Pi_{r,\parallel} = a_{\parallel}$

- ❑ **Residual stress is one component of the $\Pi_{r,\parallel}$, so enters via its divergence. Surface force—acting on the surface of fluid element**
- ❑ **Turbulent acceleration, $a_{\parallel}$ as a source/sink. Volume force—acting on everywhere of the fluid element**



→ **Surface force:** $F = -\int_{r_1}^{r_2} \nabla \cdot \Pi_{r,\parallel} = -\Pi_{r,\parallel}\big|_{r_1}^{r_2}$

→ **Volume force:** $F = \int_{r_1}^{r_2} a_{\parallel}$

Can we write $a_{\parallel}$ in terms of a divergence of vector (may plus a curl of another vector) which has clear physical meaning?

Turbulent acceleration vs residual stress

➢ **Mean flow equation:** $\frac{\partial}{\partial t}\langle U_{\parallel}\rangle + \nabla \cdot \Pi_{r,\parallel} = a_{\parallel}$

❑ **The order of magnitude :**

❑ $\Pi_{\parallel,res} = \rho_i v_{thi}^3 \frac{\hat{s}}{qR} \sum_k (\mathrm{Re}\,\tau_{ck}) k_\theta^2 x^2 \frac{\partial I_k}{\partial x}$ **[Gurcan, PoP 2010]**

❑ $\frac{a_{\parallel}}{\nabla \cdot \Pi_{\parallel,res}} \sim \frac{T_i}{T_e}\frac{L}{L_{T_i}}$, L**: scale length of** $\Pi_{\parallel,res}$

→ order of unity for $L \sim L_{T_i}$, $T_i \sim T_e$

⟹ **Turbulent acceleration is qualitatively different from, but quantitatively comparable to the divergence residual stress!**

[Lu Wang* and P.H. Diamond Phys. Rev. Lett. 110, 265006 (2013)]

Outline

Part II: turbulent acceleration and momentum conservation

- **Turbulent acceleration of parallel flow**
  - **Intrinsic flow drive by E.S. and E.M. ITG turbulence**
- **Consistency between turbulent acceleration and momentum conservation**
- **Summary**

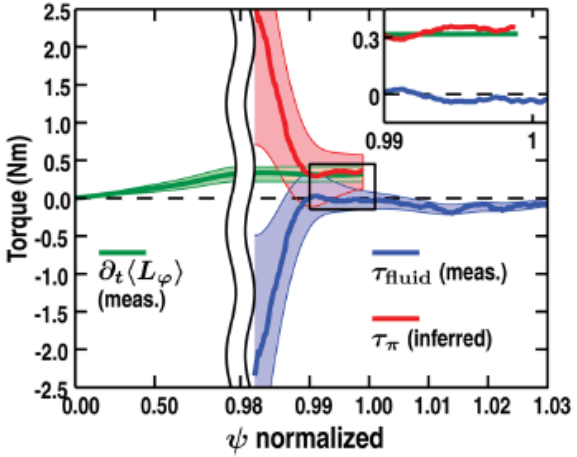
Electromagnetic effects

- For low-β plasmas → electrostatic approximation
- Regions of steep equilibrium gradients, pedestal → finite-β effects, electromagnetic contributions are important

[Scott B. *Plasma Phys. Control. Fusion* 39 1635, (1997)]

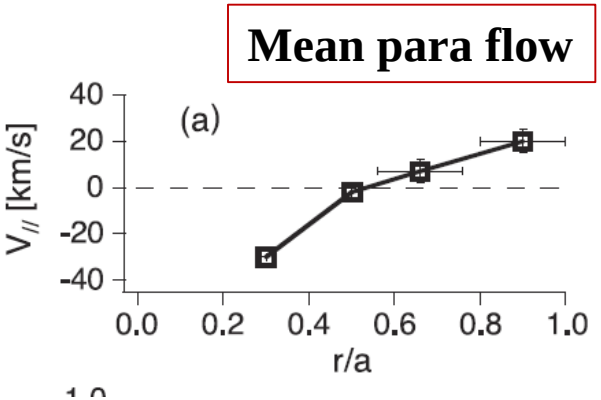
- DIII-D: The torque from fluid Reynolds stress cannot explain the edge rotation profile. Kinetic stress?

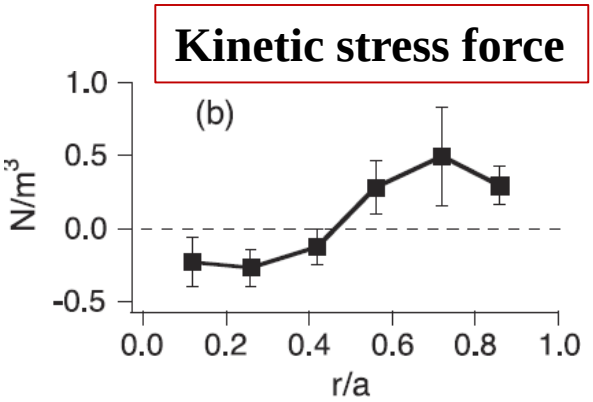
[Muller et al., PoP **18**, 072504 (2011)]



- MST: Kinetic Stress (related to magnetic fluctuation) driven intrinsic flow has been observed in recent experiment on MST

[W.X. Ding et al., PRL **110**, 065008 (2013)]





Parallel flow in E.M. turbulence

➢ **Mean parallel flow velocity equation from E.M. GK equation**

$$\frac{\partial\langle U_\parallel\rangle}{\partial t}+\nabla\cdot\langle\delta\mathbf{v}_{E\times B,\boldsymbol{r}}\delta U_\parallel\rangle+\frac{1}{m_i n_0}\nabla\cdot\langle\boldsymbol{\delta\hat{B}_r}\delta P_i\rangle+\frac{e}{m_i}\nabla\cdot\langle\boldsymbol{\delta\hat{B}_r}\delta\phi\rangle$$
$$=\frac{1}{m_i n_0}\langle\delta\hat{n}\hat{\boldsymbol{b}}\bullet\nabla\delta P_i\rangle+\frac{1}{m_i n_0}\langle\delta\hat{n}\boldsymbol{\delta\hat{B}}_r\rangle\cdot\nabla P_i$$

✓ **Turbulent stresses: enter the equation by their divergence-surface force**

❑ **Kinetic stress** $\Pi^{\mathrm{kin}}=\frac{1}{m_i n_0}\langle\boldsymbol{\delta\hat{B}_r}\delta P_i\rangle$,

❑ **Cross Maxwell stress** $\Pi^{\mathrm{Max}}=\frac{e}{m_i}\langle\boldsymbol{\delta\hat{B}_r}\delta\phi\rangle$

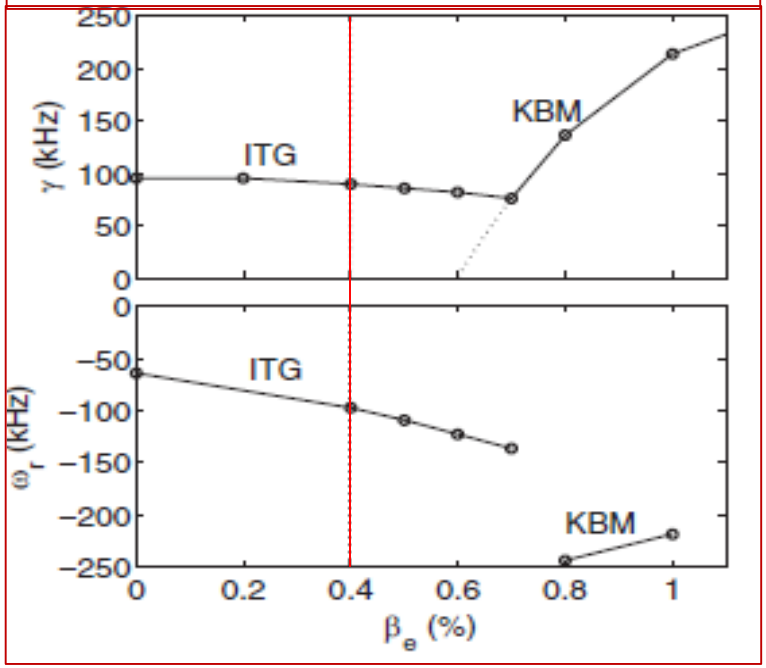
✓ **Turbulent acceleration: source or sink-body force**

❑ $a_M=\frac{1}{m_i n_0}\langle\delta\hat{n}\boldsymbol{\delta\hat{B}}_r\rangle\cdot\nabla P_i$, **new E.M. turbulent acceleration**

➢ **All terms directly related to magnetic fluctuation (Π^{kin}, Π^{Max}, a_M) can drive intrinsic flow. Π^{Rey} and $\boldsymbol{a_P}$ also bear E.M effects through E.M. effects on $\delta U_\parallel$ and δP_i.**

Intrinsic parallel flow drive by E.M. ITG

GK Simulation : E.M. ITG dominant in DIII-D pedestal top by GTC [Holod et al., NF 2015]

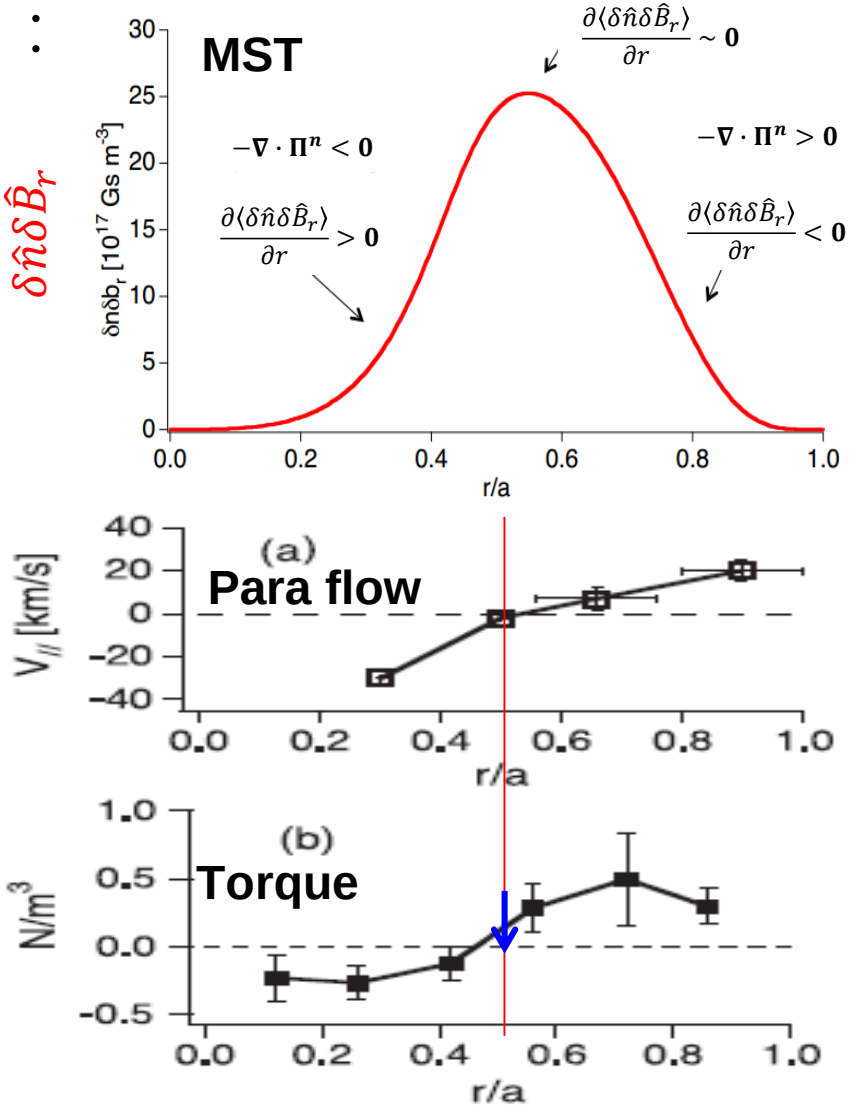


Our theoretical results using the same parameters: [Shuitao Peng and Lu Wang*, NF 57 036003 (2017)]

Intrinsic flow drive	Non-resonant part	Resonant part
Residual stress force	$\mp\frac{1}{L}\sum_k \Pi^{NR}_{ES,k}\left(1-4\hat{\beta}_e\right)$	$\pm\frac{1}{L}\sum_k \Pi^{Res}_{ES,k}\frac{43}{4}\left(1+\frac{141}{86}\hat{\beta}_e\right)$
Kinetic stress force	$\mp\frac{1}{L}\sum_k \Pi^{NR}_{ES,k}\left(-20\hat{\beta}_e\right)$	$\pm\frac{1}{L}\sum_k \Pi^{Res}_{ES,k}\left(\frac{43}{4}*\frac{3}{2}\hat{\beta}_e\right)$
Cross Maxwell stress force	$\mp\frac{1}{L}\sum_k \Pi^{NR}_{ES,k}\left(\hat{\beta}_e\right)$	–
Total stress force	$\mp\frac{1}{L}\sum_k \Pi^{NR}_{ES,k}\left(1-23\hat{\beta}_e\right)$	$\pm\frac{1}{L}\sum_k \Pi^{Res}_{ES,k}\frac{43}{4}\left(1+\frac{135}{43}\hat{\beta}_e\right)$
Turbulent acceleration a_p	$\frac{1}{L_n}\sum_k \Pi^{NR}_{ES,k}\,16\left(1+\frac{21}{4}\hat{\beta}_e\right)$	$\frac{1}{L_n}\sum_k \Pi^{Res}_{ES,k}\frac{43}{2}\left(1+\frac{387}{172}\hat{\beta}_e\right)$
Turbulent acceleration a_M	$\frac{1}{L_n}\sum_k \Pi^{NR}_{ES,k}\,16\left(-\hat{\beta}_e\right)$	–
Total turbulent acceleration $a_\parallel$	$\frac{1}{L_n}\sum_k \Pi^{NR}_{ES,k}\,16\left(1+\frac{17}{4}\hat{\beta}_e\right)$	$\frac{1}{L_n}\sum_k \Pi^{Res}_{ES,k}\frac{43}{2}\left(1+\frac{387}{172}\hat{\beta}_e\right)$

Note: $\Pi^{NR}_{ES,k}=\rho_s c_s^3\frac{|\gamma_k|}{\omega_r^2}k_\theta k_{||}\left|\delta\hat{\phi}_k\right|^2$; $\Pi^{Res}_{ES,k}=\frac{1}{2}\sqrt{\frac{2\pi}{\tau}}\rho_s c_s^2 k_\theta\frac{k_\parallel}{|k_\parallel|}\zeta^2 e^{-\frac{\zeta^2}{2}}\left|\delta\hat{\phi}_k\right|^2$. These results are obtained in the limits $\omega_r\approx-2\omega_{*e}$, $k_\parallel\approx\frac{1}{qR_0}$, $k_\theta\rho_s\approx 0.17$ and $k_\perp^2=2k_\theta^2$ for typical parameters of the pedestal top region on DIII-D [28]. $\hat{\beta}_e\approx\beta_e\frac{q^2R_0^2}{L_n^2}=0.144$

Implications on MST experiments



- **Kinetic stress $\langle\delta\hat{n}\delta\hat{B}_r\rangle$ is measured ($\delta\hat{T}_i$ is difficult to measure from EXP.)**
- **E.M. turbulent acceleration $a_M = \frac{1}{m_i n_o}\langle\delta\hat{n}\delta\hat{B}_r\rangle\frac{\partial}{\partial r}P_i$ can be also obtained if pressure profile available**
- **For this case, $\langle\delta\hat{n}\delta\hat{B}_r\rangle > 0$, and if $\frac{\partial}{\partial r}P_i < 0$**

→ $a_M < 0$, i.e., contributes a ctr-torque

→May explain the data of parallel flow and intrinsic torque more consistently?

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- Turbulent acceleration of parallel flow
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Existence of turbulent acceleration

➢ **Another method to derive the mean parallel flow:**

(1) $$\frac{\partial\langle U_\parallel\rangle}{\partial t}=\frac{1}{\langle n\rangle}\left[\frac{\partial\langle nU_\parallel\rangle}{\partial t}-\langle U_\parallel\rangle\frac{\partial\langle n\rangle}{\partial t}-\langle\delta U_\parallel\frac{\partial}{\partial t}\delta n\rangle-\langle\delta n\frac{\partial}{\partial t}\delta U_\parallel\rangle\right]\leftrightarrow\frac{\partial}{\partial t}\langle U_\parallel\rangle=\left\langle\frac{1}{n}\left[\frac{\partial}{\partial t}\left(nU_\parallel\right)-U_\parallel\frac{\partial}{\partial t}n\right]\right\rangle$$

with the mean parallel momentum density equation

(2) $$\frac{\partial\langle nU_\parallel\rangle}{\partial t}=-\nabla\cdot\langle\delta\mathbf{v}_{E\times B,r}\delta\left(nU_\parallel\right)\rangle-\frac{1}{m_i}\nabla\cdot\langle\boldsymbol{\delta\widehat{B}_r}\delta P_i\rangle-\frac{e}{m_i}n_0\nabla\cdot\langle\boldsymbol{\delta\widehat{B}_r}\delta\phi\rangle$$
$$-\frac{e}{m_i}\langle\delta n\left(\widehat{\boldsymbol{b}}\cdot\nabla\delta\phi+\frac{1}{c}\frac{\partial\delta A_\parallel}{\partial t}\right)\rangle$$

Cancelled!

and mean density equation

(3) $$\frac{\partial\langle n\rangle}{\partial t}=-\nabla\cdot\langle\delta\mathbf{v}_{E\times B,r}\delta n\rangle-\nabla\cdot\langle\boldsymbol{\delta\widehat{B}_r}\delta\left(nU_\parallel\right)\rangle$$

Linearizing density equation and parallel flow velocity equation to obtain

(4) $$\langle\delta U_\parallel\frac{\partial}{\partial t}\delta n\rangle=-\langle\delta U_\parallel\nabla\cdot\left(\delta\mathbf{v}_{E\times B}n_0\right)\rangle-\langle\delta U_\parallel\nabla\cdot\left(n_0\langle U_\parallel\rangle\boldsymbol{\delta\widehat{B}}_\perp\right)\rangle-\langle\delta U_\parallel\nabla\cdot\left(\delta\left(nU_\parallel\right)\widehat{\boldsymbol{b}}\right)\rangle$$

(5) $$\langle\delta n\frac{\partial}{\partial t}\delta U_\parallel\rangle=-\langle\delta n\nabla\cdot\left(\delta\mathbf{v}_{E\times B}\langle U_\parallel\rangle\right)\rangle-\frac{e}{m_i}\langle\delta n\left(\widehat{\boldsymbol{b}}\cdot\nabla\delta\phi+\frac{1}{c}\frac{\partial\delta A_\parallel}{\partial t}\right)\rangle-\frac{1}{m_in_o}\left[\langle\delta\hat{n}\widehat{\boldsymbol{b}}\cdot\nabla\delta P_i\rangle+\langle\delta\hat{n}\boldsymbol{\delta\widehat{B}_r}\rangle\cdot\nabla P_i\right]$$

→ **Eqs. (1)-(5) also yield a mean parallel flow velocity equation**

Existence of turbulent acceleration

➢ **Two methods to derive the mean parallel flow:**

✓ **I:** $\frac{\partial}{\partial t}\langle U_\parallel \rangle = \left\langle \frac{1}{n}\left[\frac{\partial}{\partial t}(nU_\parallel) - U_\parallel \frac{\partial}{\partial t} n\right]\right\rangle$

✓ **II:** $\frac{\partial \langle U_\parallel \rangle}{\partial t} = \frac{1}{\langle n \rangle}\left[\frac{\partial \langle nU_\parallel \rangle}{\partial t} - \langle U_\parallel \rangle \frac{\partial \langle n \rangle}{\partial t} - \langle \delta U_\parallel \frac{\partial}{\partial t}\delta n \rangle - \langle \delta n \frac{\partial}{\partial t}\delta U_\parallel \rangle\right]$

➔ **The SAME mean parallel flow velocity equation!**

$$\frac{\partial \langle U_\parallel \rangle}{\partial t} + \nabla \cdot \langle \delta \mathbf{v}_{E\times B,\boldsymbol{r}} \delta U_\parallel \rangle + \frac{1}{m_i n_0} \nabla \cdot \langle \boldsymbol{\delta \hat{B}_r} \delta P_i \rangle + \frac{e}{m_i} \nabla \cdot \langle \boldsymbol{\delta \hat{B}_r} \delta \phi \rangle$$
$$= \frac{1}{m_i n_0} \langle \delta \hat{n} \hat{\boldsymbol{b}} \bullet \nabla \delta P_i \rangle + \frac{1}{m_i n_0} \langle \delta \hat{n} \boldsymbol{\delta \hat{B}}_r \rangle \cdot \nabla P_i$$

❑ **The turbulent acceleration DOSE exist.**

❑ **It is NOT because of simply regrouping terms!**

Momentum conservation

- One frequently asked question: Dose the turbulent acceleration of parallel flow contradict momentum conservation?
- Answer: NO!

- Ion flow $\langle U_{\parallel} \rangle$ and ion kinematic momentum $\langle nU_{\parallel} \rangle$ are different concepts! The later one is strongly coupled with density.
- The conserved quantity can be either total parallel canonical momentum carried by both species or total parallel momentum carried by particles and field but not the ion kinematic momentum $\langle nU_{\parallel} \rangle$ or ion flow $\langle U_{\parallel} \rangle$.
- Turbulent acceleration DOES NOT contradict momentum conservation!
- We can also obtain the momentum conservation by summing momentum equation over species.

Momentum conservation

- **Two equivalent momentum conservation equations:** $\frac{\partial A}{\partial t} + \nabla \cdot S = 0$

I. Canonical momentum conservation:

$$\frac{\partial}{\partial t}\sum_s \langle p_{\parallel s}\rangle + \nabla \cdot \langle \delta \mathbf{v}_{E\times B.r} m_i n_i U_{\parallel i} + \delta \widehat{\boldsymbol{B}}_r P - \frac{c^2 m_i n_i}{B^2}\nabla_r \delta\phi \widehat{\boldsymbol{b}} \cdot \nabla\delta\phi\rangle = 0$$

✓**Conserved quantity:** $\sum_s \langle p_{\parallel s}\rangle = \langle m_i n_i U_{\parallel i} - n_{pol}\frac{e}{c}\delta A_{\parallel}\rangle$

Kinematic momentum **Magnetic vector momentum**

II. Total momentum (particle + E.M. field) conservation:

$$\frac{\partial}{\partial t}\langle m_i n_i U_{\parallel i} + m_i n_i \frac{c^2}{B^2}(\delta \boldsymbol{E} \times \delta \boldsymbol{B})_{\parallel}\rangle$$

$$+\nabla \cdot \langle \delta \mathbf{v}_{E\times B.r} m_i n_i U_{\parallel i} + \delta \widehat{\boldsymbol{B}}_r P - \frac{c^2 m_i n_i}{B^2}\nabla_r \delta\phi \left(\widehat{\boldsymbol{b}} \cdot \nabla\delta\phi + \frac{1}{c}\frac{\partial \delta A_{\parallel}}{\partial t}\right)\rangle = \mathbf{0}$$

✓**Conserved quantity:** $\langle m_i n_i U_{\parallel i} + m_i n_i \frac{c^2}{B^2}(\delta \boldsymbol{E} \times \delta \boldsymbol{B})_{\parallel}\rangle$

Particle **E.M. field**

$$\boldsymbol{g}_F = \delta \boldsymbol{D} \times \delta \boldsymbol{B}, \quad \text{where} \quad \delta \boldsymbol{D} = \epsilon \delta \boldsymbol{E}$$
$$\epsilon = m_i n_0 \frac{c^2}{B^2}$$

[Shuitao Peng and Lu Wang*, Phys. Plasmas 24, 012304 (2017)]

Summary & Discussions

- **Turbulent acceleration can also act as a new mechanism for intrinsic rotation drive**
- **E. M. effects on intrinsic rotation in the edge pedestal are important**
- **Turbulent acceleration DOES NOT contradict momentum conservation!**

- **Discussions: Turbulent electron momentum transport → Intrinsic current [He, Wang and Zhuang, NF 2018]**

  → Interaction between turbulence and MHD

  - ✓ **Intrinsic current effects on NTM**
  - ✓ **Turbulence modulation by NTM**

Thanks for your attention!